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Stochastic Methods Applied to Power System Operations with Renewable Energy: A Review

Energy Systems Division

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by
Z. Zhou,[1] C. Liu,[1,2] and A. Botterud[1]
[1]Argonne National Laboratory
[2]Electric Reliability Council of Texas (ERCOT)

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CONTENTS

ACKNOWLEDGMENTS v
ACRONYMS vi
EXECUTIVE SUMMARY 1
1 INTRODUCTION 5
2 POWER SYSTEMS, ELECTRICITY MARKETS, AND RENEWABLE ENERGY 7
2.1 Power System Operations 7
2.2 Introduction to Unit Commitment and Economic Dispatch 8
2.3 Electricity Markets 9
2.4 Impacts of Renewable Energy 11
2.5 Modeling Power System Scheduling and Dispatch with Renewable Energy 12
3 ADDRESSING UNCERTAINTY AND VARIABILITY WITH OPERATING RESERVES 15
3.1 Overview 15
3.2 Probabilistic Evaluation of Reserves in Thermal Systems 16
3.3 Probabilistic Evaluation of Reserves with Renewable Energy 17
3.4 Operating Reserves in Integration Studies and Industry 18
3.5 Stochastic Scheduling Formulations with Implicit Reserves 20
3.6 Summary 21
4 STOCHASTIC PROGRAMMING FORMULATIONS OF THE UNIT COMMITMENT PROBLEM 23
4.1 Overview 23
4.2 A General and Specific Stochastic Unit Commitment Formulation 24
4.3 Review of Stochastic Programming Unit Commitment Formulations 26
4.4 Uncertainty Modeling 28
4.4.1 Probabilistic Forecasting 29
4.4.2 Scenario Generation 30
4.4.3 Scenario Reduction 31
4.5 Modeling Tools and Solution Algorithms 31
4.6 Performance Comparison Metrics 32
4.7 Test Beds 33
4.8 Real-world System Case Studies and Industrial Applications 33
4.9 Summary 34
5 OTHER UNIT COMMITMENT FORMULATIONS UNDER UNCERTAINTY 38
5.1 Overview 38
5.2 Chance-constrained Optimization Based Unit Commitment 39
5.3 Robust Optimization Based Unit Commitment 39
5.3.1 Diverse Robust Unit Commitment Formulations 39
5.3.2 Solution Methods and Uncertainty Sets 41
5.3.3 Test Bed 41

CONTENTS (CONT.)

5.4 Interval Optimization Based Unit Commitment....42
5.5 Fuzzy Set Based Unit Commitment....44
5.6 Summary....44

6 CONCLUSIONS....47

7 REFERENCES....49

FIGURES

1 Overview of Issues in Power System Operations and Control....7

2 Regions in North America with Electricity Markets Operated by ISOs or RTOs....10

3 Main Stages in Electricity Market Operations with Day-Ahead and Real-Time Markets for Energy and Operating Reserves....10

4 Typical Operating Reserve Products Traded in Current Electricity Markets in the United States....12

TABLES

1 Summary of Current and Proposed Scheduling Approaches with Renewable Energy....2

2 Summary of Operating Reserve Approaches, Accounting for Uncertainty....21

3 Summary of Stochastic UC Formulations....35

4 Solution Methods for Stochastic UC Problems....36

5 Summary of Probabilistic Wind Power Forecasting Methods....36

6 Summary of Cost Savings of Stochastic UC Methods Compared to Deterministic Benchmark....37

7 Summary of Other Unit Commitment Formulations Under Uncertainty....45

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ACRONYMS

CAISO	California Independent System Operator
CPS2	Control Performance Standard 2
ED	economic dispatch
EENS	Expected Energy Not Served
ERCOT	Electric Reliability Council of Texas
FERC	Federal Energy Regulatory Commission
ISO	Independent System Operator
LMP	locational marginal price
LOLP	loss-of-load probability
MISO	Midcontinent Independent System Operator
NWP	numerical weather prediction
ORDC	operating reserve demand curve
pdf	probability density function
RTO	Regional Transmission Organization
SAA	sample average approximation
SDP	stochastic dynamic programming
UC	unit commitment
VOLL	value of lost load

EXECUTIVE SUMMARY

Renewable energy resources have been rapidly integrated into power systems in many parts of the world, contributing to a cleaner and more sustainable supply of electricity. Wind and solar resources also introduce new challenges for system operations and planning in terms of economics and reliability because of their variability and uncertainty. Operational strategies based on stochastic optimization have been developed recently to address these challenges. In general terms, these stochastic strategies either embed uncertainties into the scheduling formulations (e.g., the unit commitment [UC] problem) in probabilistic forms or develop more appropriate operating reserve strategies to take advantage of advanced forecasting techniques. Other approaches to address uncertainty are also proposed, where operational feasibility is ensured within an uncertainty set of forecasting intervals.

In this report, a comprehensive review is conducted to present the state of the art through Spring 2015 in the area of stochastic methods applied to power system operations with high penetration of renewable energy. Chapters 1 and 2 give a brief introduction and overview of power system and electricity market operations, as well as the impact of renewable energy and how this impact is typically considered in modeling tools. Chapter 3 reviews relevant literature on operating reserves and specifically probabilistic methods to estimate the need for system reserve requirements. Chapter 4 looks at stochastic programming formulations of the UC and economic dispatch (ED) problems, highlighting benefits reported in the literature as well as recent industry developments. Chapter 5 briefly introduces alternative formulations of UC under uncertainty, such as robust, chance-constrained, and interval programming. Finally, in Chapter 6, we conclude with the main observations from our review and important directions for future work.

An overview of the main operational strategies reviewed in this report is provided in Table 1. Compared to current deterministic scheduling and dispatch strategies, the more recent approaches focus on improved representation of uncertainty in the problem formulation, either through probabilistic descriptions, as in stochastic programming models, or through various types of uncertainty sets that do not necessarily assume an underlying probability distribution. The methods also differ in terms of whether and how uncertainty is represented in the objective function, computational burden, and impacts on the calculation of market prices. A common challenge across most of these methods is to generate an adequate quantitative uncertainty representation that reflects the real-world probabilistic nature of renewable resource availability.

Table 1 indicates qualitative advantages and disadvantages of the different scheduling methods. However, very limited research has been done on consistently comparing quantitative results obtained across all the different operational methods. Objective and realistic performance assessments are also hard to do, in part because methods are based on different modeling assumptions, but also because all real-world complexities of power system operations cannot be reflected in operational simulations. Although one has to be careful in drawing firm conclusions, there is no doubt that the research literature has indicated that new operational practices for dealing with uncertainty in renewable resources have the potential to provide substantial benefits.

TABLE 1 Summary of Current and Proposed Scheduling Approaches with Renewable Energy

Optimization Method	Objective Function	Uncertainty Representation	Operating Reserves	Pros/Cons
Deterministic programming w/fixed reserves	Min Cost	None	Fixed reserve constraints	+ Current practice with established models + Well-defined prices for energy and reserves - Uncertainty not explicitly represented - Forecast uncertainty not reflected in reserve requirements
Deterministic programming w/dynamic reserves	Min Cost	Only through reserve constraints	Dynamic reserve constraints	+ Small departure from current practice + Well-defined prices for energy and reserves + Reserve requirements reflect forecast uncertainty - Uncertainty not explicitly represented
Stochastic programming	Min Expected Cost	Scenarios/ scenario trees	Implicit*	+ Rational decision strategy + Minimizes expected cost + Explicit uncertainty representation - Computational burden - Adequate scenario generation - Complex energy prices, no explicit reserve prices
Chance-constrained programming	Min Cost for baseline	Random variables, usually with parametric probability distributions	Implicit*	+ Explicit uncertainty representation + Ensures target reliability level - Uncertainty not reflected in objective function - Adequate uncertainty representation - No explicit reserve prices
Robust programming	Min Max Cost	Uncertainty set without assumptions on probability	Implicit*	+ Explicit uncertainty representation - Conservative decision strategy - Hard to derive adequate uncertainty set - Complex energy and reserve prices
Interval programming	Min Cost for Baseline	Continuous interval set without assumptions on probability	Implicit*	+ Explicit uncertainty representation + Can provide cost intervals + Fast computation - Uncertainty not reflected in objective function - Hard to derive adequate uncertainty set - No explicit reserve prices
Fuzzy programming	Min Cost Membership	Fuzzy possibility set	Implicit*	+ Explicit uncertainty representation - Uncertainty not reflected in objective function - Hard to derive adequate uncertainty set - No explicit reserve prices

* Implicit reserves may be complemented by explicit reserve requirements, accounting for uncertainty and variability not captured in the mathematical formulation and uncertainty description.

The main findings from the literature review in this report can be summarized as follows:

- Probabilistic algorithms have been proposed and applied for a long time to estimate the need for operating reserves, accounting for the stochastic nature of transmission and generation outages. Still, deterministic operating rules based on heuristics (e.g., the single largest contingency rule for reserves) are often applied in practice.

- In recent years, there has been a surge in research on the application of stochastic methods for power system operations with high penetration of renewable energy. The stochastic UC problem has received most of the attention among researchers, but new approaches for probabilistic estimates of operating reserves have also been proposed.

- Although the majority of research has focused on stochastic programming formulations of the UC problem, alternative UC formulations under uncertainty, such as robust, chance-constrained, and interval programming are also gaining popularity.

- The main focus in the literature on stochastic scheduling and dispatch is on cost savings and reliability impacts, including metrics such as expected operating cost, committed thermal unit capacity, scheduled operating reserve capacity, renewable energy utilization, number of thermal unit startups, CO_2 emissions, and load curtailment. However, there is also increasing attention to market implications, such as pricing of energy and reserves.

- Most of the studies report operating cost savings from using stochastic formulations, especially if potential savings from cost penalties for load or reserve curtailment are considered. There are also reports indicating that scenario-based stochastic UC formulations may lead to more frequent start-ups of thermal units. More specifically, while there are fewer start-ups of base load units, more start-ups occur for middle-merit natural gas-fired units. The total number of start-ups is slightly higher to account for the variability of wind power supply.

- There are no standard frameworks or metrics to compare the pros and cons of different operational strategies. Except for using some standard IEEE test power systems (e.g., the IEEE 118-bus test system) for evaluation purposes, the formulations proposed in the current literature have many different features (for example, demand response, storage, and emission constraints), which make it difficult to compare the reported performance for operational strategies across different studies. Moreover, the reference or benchmark strategy often varies in different papers. In some studies, deterministic operating strategies are used as a benchmark, while in some other studies alternative stochastic formulations are used as a benchmark.

- Industry adoption of probabilistic methods for operational decisions is still limited, but there is increasing interest in the topic. So far, in the United States, studies have been reported by two independent system operators (ISOs), ISO New England and Midcontinent ISO, to test the applications of stochastic methods for operational decision-making on the large-scale systems they operate.

Directions for future work include the following:

- More systematic testing and comparison of different operational strategies, accounting for a larger set of the real-world issues, constraints, and potential future regulatory policies in power system and electricity market operations.

- A closer investigation of the interaction between explicit operating reserve requirements imposed by traditional reserve constraints and the implicit reserves provided by stochastic scheduling and dispatch formulations.

- Further investigation of the potential implications for pricing and market incentives under stochastic UC and ED, with the goal of providing efficient signals for operations and investments for all market participants.

- Further refinements of methods for probabilistic forecasting and scenario generation and reduction, as critical inputs to stochastic methods for power system operations.

- Further investigation of interactions between stochastic short-term operations and risk-constrained long-term planning decisions.

- Development of stochastic methods for mid-term operation and coordination, such as maintenance scheduling from the system operator's point of view, as well as fuel and emissions planning from generation companies' perspectives.

- Testing on real-world and large-scale systems, with engagement from utility companies and system operators to validate the performance of stochastic methods and provide better quantitative estimates of benefits. Industry feedback and suggestions for improvements in research-grade algorithms are critical to developing the industrial tools needed for more economical and reliable power system operations with large shares of renewable energy.

1 INTRODUCTION

The rapid expansion of variable renewable energy resources, such as wind and solar power, into the electric power grid gives rise to new challenges in power system operations. At the heart of the challenge is to efficiently address the uncertainty and variability of the renewable resources in operational decisions. In this report, we review literature on potential applications of probabilistic methods in power system operations with renewable energy, with a particular focus on unit commitment (UC) and economic dispatch (ED) methods from day-ahead scheduling to real-time operations. Although research has been conducted on this topic for a long period of time, relatively few industry applications have emerged so far. However, there is increasing interest in industry adoption of such methods in many parts of the world, as the penetration levels of renewable resources continue to rise. In a companion report [1], we do a quantitative comparison of different scheduling strategies, focusing primarily on deterministic, stochastic, and interval formulations of the UC problem.

The literature on power system operations with renewable energy has grown rapidly in recent years. In this report, we have attempted to include important references related to system scheduling and dispatch under uncertainty in renewable energy. However, the review is not exhaustive, and there is certainly relevant literature that is not covered in the report. Moreover, note that most of the literature review took place in Fall 2014 and Spring 2015. More recent literature is therefore not included in this report.

Optimal generation scheduling decisions in the power grid have been researched extensively for decades (e.g., Cohen and Sherkat 1987 [2]). In particular, the UC problem has received extensive attention (e.g., Hobbs et al. 2001 [3], Padhy 2004 [4]) because of its complex non-convex mathematical structure and its importance for power system operations. Meanwhile, a lot of research has been done in recent years on integration of renewable resources into the power grid (Milligan et al. 2002 [5], Smith et al. 2007 [6], Xie et al. 2011 [7], Holttinen et al. 2013a [8], and Bessa et al. 2014 [9] provide overviews of some of these efforts). The application of stochastic methods in power system operations is frequently identified as one potential solution to address the corresponding uncertainty challenges more efficiently. At present, there is increasing interest in stochastic programming formulations of the UC problem, owing, in part, to advances in computation that make it possible to solve such problems faster than in the past, but also motivated by the increase in wind and solar generation. Recent reviews of the stochastic UC problem are provided by Tahanan et al. 2014 [10], Dai et al. 2015 [11], and Zheng et al. 2015 [12].

This report focuses specifically on the use of stochastic methods in power system and electricity market operations with renewable energy. In addition to reviewing recent advances in stochastic formulations of the UC and ED problems, we also look at the closely related topic of probabilistic estimation of operating reserves. Moreover, we briefly discuss alternative approaches to address the uncertainty from wind and solar resources in operational decisions. We review reported benefits of stochastic methods and also discuss current efforts by the United States utility industry within these domains. A number of other potential applications of

probabilistic methods, such as hydrothermal scheduling, renewable generation bidding and risk management, and long-term generation and transmission planning, are left out of the discussion.

The report has the following structure: Chapter 2 gives a brief overview of power system and electricity market operations, as well the impact of renewable energy and how this impact is typically considered in modeling tools. Chapter 3 reviews relevant literature on operating reserves and specifically probabilistic methods to estimate the need for system reserve requirements. Chapter 4 looks at stochastic programming formulations of the UC and ED problems, highlighting benefits reported in the literature as well as recent industry developments. Chapter 5 briefly introduces alternative formulations of UC under uncertainty, such as robust and chance-constrained programming. Finally, we conclude by stating the main observations from our review and important directions for future work.

2 POWER SYSTEMS, ELECTRICITY MARKETS, AND RENEWABLE ENERGY

In this chapter, we give a brief introduction to power system operations, electricity markets, and the impact of renewable energy on the underlying decision processes to manage supply and demand in the power grid. We conclude with a brief discussion of how to model scheduling and dispatch decisions in power systems with renewable energy, with a focus on traditional deterministic approaches.

2.1 POWER SYSTEM OPERATIONS

The electric power grid is a very complex engineering system, where generation must be balanced continuously with loads to maintain frequency and stability. A number of different control and operational problems must be addressed towards this end in time frames ranging from microseconds to days (Figure 1). In the very short term, grid harmonics and stability are addressed through system control and automated response actions. In the intermediate time frame, operating reserve and energy dispatch are employed to maintain system frequency and balance supply and demand. At longer time scales, the challenge is to schedule sufficient resources to handle variability and uncertainty in the load and electricity supply resources. Increasing amounts of renewable energy add to the existing uncertainty and variability in the system, and the impacts are typically most significant in the middle range of the operational time frame, as indicated in Figure 1.

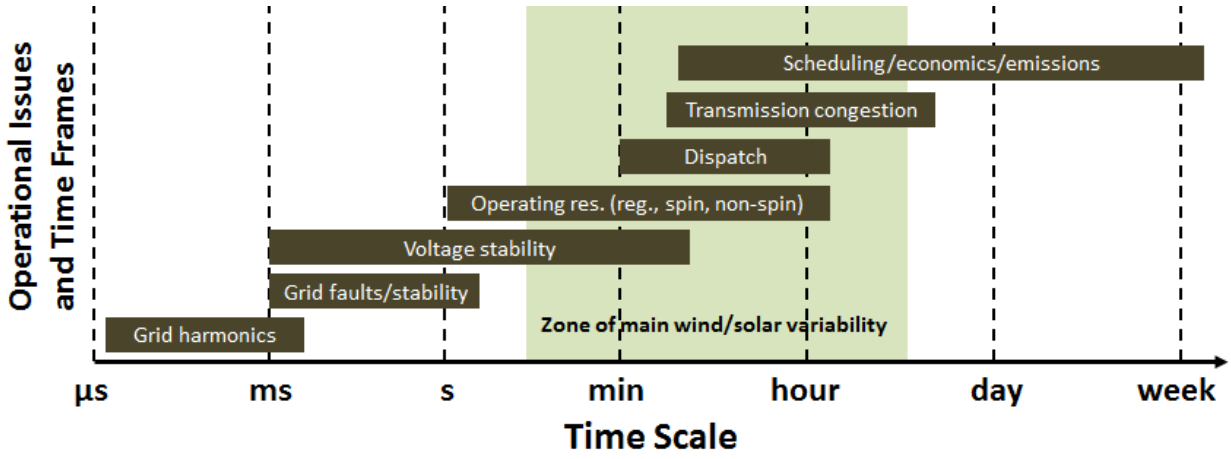

FIGURE 1 Overview of Issues in Power System Operations and Control (Source: Botterud et al. 2014 [13]).

2.2 INTRODUCTION TO UNIT COMMITMENT AND ECONOMIC DISPATCH

UC and ED are at the heart of power system operations. The UC problem is defined as the decision for scheduling a set of generating units to be on, off, or in start-up, shutdown, or standby mode for a defined period of time to meet a certain objective. The objective varies depending on the applications. For a power system operated by a vertically integrated utility company or by an independent system operator, the objective is to minimize the total production cost subject to meeting all demand and reserve requirements within the geographic footprint. In a market environment, the objective of a generation company is to maximize its profit by scheduling its available generation units subject to its contracts and bids into the electricity market. In both cases, the scheduling decisions are subject to physical constraints of generation technologies, including ramping rates, start-up and shutdown time period, etc.

The ED problem is defined as the scheduling of production of a set of online units to satisfy a specific demand and other system operating constraints with its total production cost minimized or profit maximized. The set of online generation units is usually the result of solving the UC problem for the same system. The output of the ED problem includes the amount of generation dispatched from each on-line unit at each time step. The prices of energy and reserves at each time step at each location can also be derived from the ED problem.

In power system operations, the results of UC and ED depend on forecasts of load and renewable resources. The closer to the operating time, the more accurate the forecast is. Slow-start units must be scheduled many hours before they can generate electricity. In contrast, fast-start units can be scheduled to produce electricity within a couple of hours. Therefore, there are usually multiple rounds of UC calculations to schedule generation units with updated forecasting information as the operating period approaches. For the UC of a given day, the earliest scheduling can be as early as one week ahead to determine the use of slow units (e.g., nuclear plants), whereas UC adjustment decisions may be made less than an hour ahead of operations.

The time resolution of the UC problem is usually one hour, although a finer time resolution may be used close to operations. The time horizon is usually at least 24 hours, to account for all the temporal constraints in the system. For the ED problem, since it only determines the output of online generation units, there are fewer temporal constraints. The time resolution may be as short as 5 minutes, and the time horizon is usually shorter than for the UC problem.

Traditionally, in power system operations, the major uncertainties are load forecasting errors and generation unit contingencies. Additional generation capacity, i.e., in terms of operating reserves, is reserved to hedge against this uncertainty. The estimation of load demand depends on the accuracy of load forecasting methods, whose error is in the range of 1%–2% [14]. Owing to the use of binary variables to represent the on/off status of each generating unit, the UC problem can be formulated as a deterministic mixed integration programming problem, where uncertainty is addressed by maintaining a certain amount of operating reserves. The resulting optimization problem is NP-hard and substantial computational resources are required for larger systems. Often, several simplifications (e.g., limited representation of the transmission system) are introduced in order to solve the problem in a reasonable time.

2.3 ELECTRICITY MARKETS

At a high level, the main objective for power system operators is to ensure that demand for electricity is met and to maintain system reliability in a cost-efficient manner. This is the case regardless of whether it is a traditional regulated utility system or a restructured electricity market. However, approaches towards achieving this goal differ. For regulated utilities, the focus is on minimizing total cost, as dictated by rules enforced by regulatory agencies (e.g., the Federal Energy Regulatory Commission [FERC] and state utility commissions). In contrast, regions with restructured electricity markets focus on unbundling different parts of the system (generation, transmission, and distribution) and creating competitive markets for individual products where such markets are possible (e.g., energy, operating reserves). Today, two-thirds of electricity consumers in the United States are served by electricity markets with multiple competing market participants and an independent system operator (ISO) or regional transmission organization (RTO) in charge of operating the system (Figure 2).

The main steps in the daily operation of ISO/RTO markets are illustrated in Figure 3. At the day-ahead stage, the ISO/RTO takes bids from consumers and offers from generators and clears the market in a process that includes security-constrained UC and ED. The trend in the United States is to solve the scheduling of energy and operating reserves at the same time and in the same problem, i.e. through so-called co-optimization, to ensure efficient resource allocation and prices. Energy prices reflecting congestion are calculated for each individual bus in the transmission network (i.e., locational marginal prices or LMPs), whereas zonal prices are typically used for operating reserves. The resulting schedules and prices are communicated to the market participants. After the day-ahead market, the ISO/RTO will take actions as needed to commit additional resources if unexpected events unfold, such as higher loads or lower renewable generation than those cleared in the day-ahead market, in a process we refer to as reliability UC. Finally, the real-time market balances the system with dispatch schedules for energy and reserves, and corresponding prices, typically calculated every five minutes in current ISO/RTO markets. Note that additional security constraints are imposed on the scheduling and dispatch solutions throughout the process, e.g., to ensure that the system can withstand plausible contingencies of transmission components and generation resources. In real time, the system must operate within all physical constraints, some of which may not be fully represented in the day-ahead scheduling and market clearing (e.g., a full AC power flow representation). An elaborate scheme is in place for financial market settlements. Generators are paid at the day-ahead LMPs for the day-ahead energy schedule, whereas any deviations are settled at the real-time prices. Certain incentive and penalty schemes are also in place to ensure that market participants offer all their available resources to the day-ahead and real-time markets and follow their dispatch instructions. Other markets may also exist in addition to the energy and operating reserve markets. For instance, financial transmission rights offer the possibility of hedging against congestion in the transmission grid and corresponding differences in LMPs at various locations. Some ISO/RTOs also have capacity markets that provide an additional revenue stream to generators or demand resources to ensure capacity adequacy in the long run.

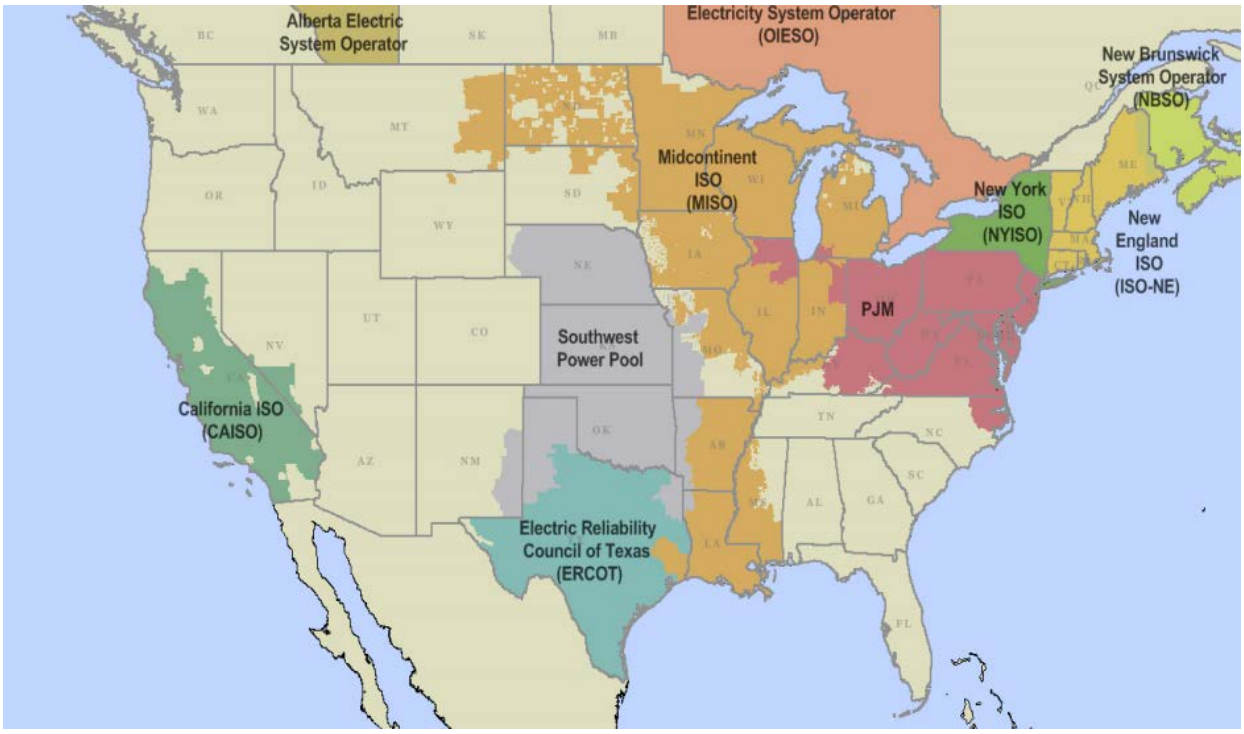

FIGURE 2 Regions in North America with Electricity Markets Operated by ISOs or RTOs (Source: FERC).

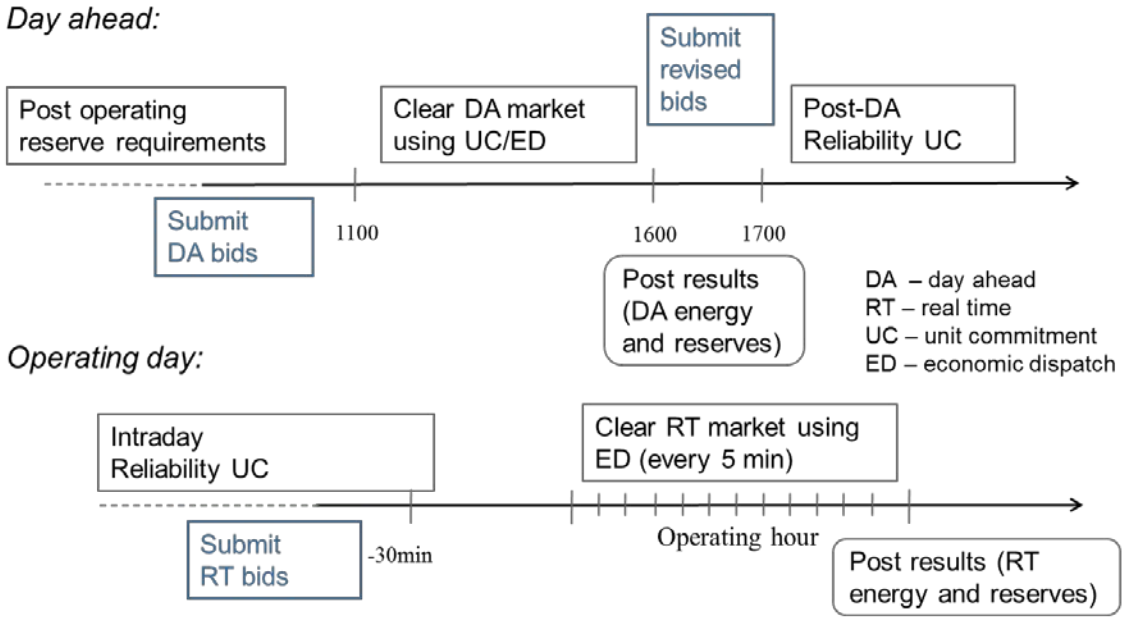

FIGURE 3 Main Stages in Electricity Market Operations with Day-Ahead and Real-Time Markets for Energy and Operating Reserves (Timeline from Midcontinent ISO [MISO]).

The basic economics of power system operations are the same in regulated and restructured areas. The lowest-cost generators are scheduled to reliably serve the expected load and then operated to meet the actual load based on security-constrained UC and ED. In regulated areas, generator marginal costs are used, while in restructured market areas, generator bid prices are used as inputs to the scheduling and dispatch process. In well-run markets, without the presence of market power, the bid-based offers should be close to marginal costs. Prices should reflect either the marginal cost of generation or the marginal utility to consumers from electricity delivery. Electricity markets make pricing and compensation for energy and other products and services more transparent than in areas with traditional utility regulation, where the focus of operations and planning is on minimizing total cost.

2.4 IMPACTS OF RENEWABLE ENERGY

The rapid increase in renewable generation significantly impacts how power systems and electricity markets are operated. In particular, the variability and forecast uncertainties in wind and solar resources create new challenges for system operators and market participants. To some extent, these challenges can be addressed by the use of forecasting at the various stages of electricity market operations (Botterud et al. 2010 [15]). However, although the accuracy of wind and solar forecasting has improved rapidly in recent years, significant forecasting errors remain because of the complex nature of these renewable resources. One consequence is the need for additional operating reserves to balance the system. Traditionally, operating reserves are defined as “That capability above firm system demand required to provide for regulation, load forecasting error, equipment forced and scheduled outages and local area protection” (NERC 2014 [16]). The most common operating reserve products traditionally scheduled and priced in United States electricity markets included regulating reserves as well as spinning and non-spinning contingency reserves. However, both California ISO (CAISO) and MISO have recently introduced an additional “flexible ramp reserve” product to address unexpected ramping events and thereby reduce the likelihood of scarcity situations and high prices in real time (Figure 4). Meanwhile, there are no markets for certain other types of reserves, such as primary frequency response. Moreover, the real-time dispatch process, especially when conducted at high frequency, also provides flexibility that is used to address some of the same balancing needs as the slower types of operating reserves. In general, the increased forecast uncertainty and variability from renewable resources will likely lead to higher reserve requirements to accommodate expected and unexpected outputs from those resources. In turn, this situation is likely to lead to higher prices for such services. The impacts of renewable energy on the needs for operating reserves, and various approaches to estimate the need for such reserves, are discussed in more detail in Chapter 3.

Renewable energy also affects the electricity markets beyond its forecasting errors and the corresponding increased need for operating reserves. For instance, existing generators may see more frequent cycling and ramping compared to their traditional operating pattern. Moreover, wind and solar power have essentially zero marginal cost, which could lead to lower energy prices on average. Moreover, increased price volatility is likely to follow from the high variability in these resources. In some situations, the energy prices may even drop to negative values because of incentive schemes for renewable energy as well as operational constraints of

other generators. These price impacts may potentially lead to revenue insufficiency and inadequate investment signals for generation as well as storage and demand technologies that contribute to balancing supply and demand in the grid (Levin and Botterud 2015 [17]). For an in-depth discussion on the general impacts of renewable energy on electricity market design, see Ela et al. (2014) [18].

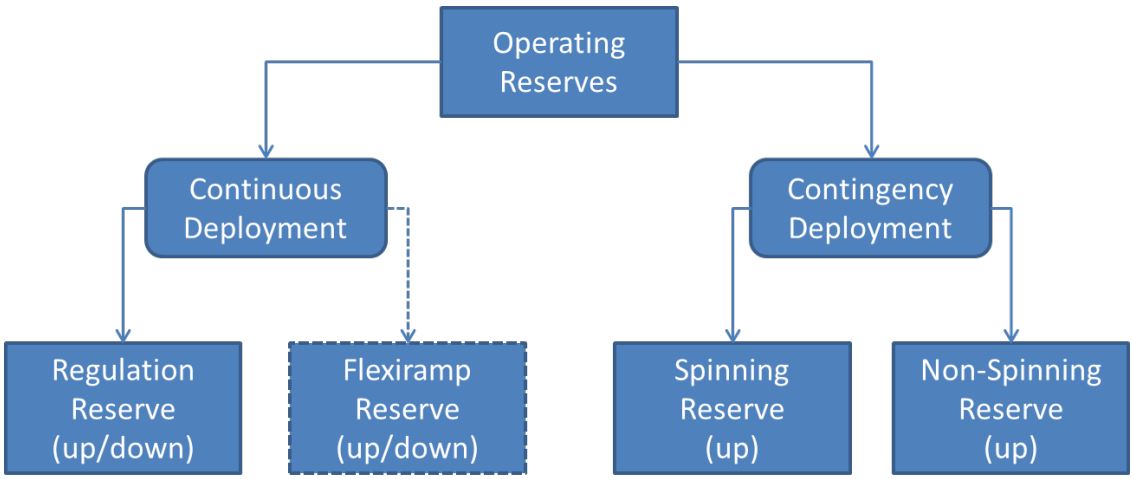

FIGURE 4 Typical Operating Reserve Products Traded in Current Electricity Markets in the United States.

2.5 MODELING POWER SYSTEM SCHEDULING AND DISPATCH WITH RENEWABLE ENERGY

Traditionally, scheduling and dispatch in power system operations have been done using deterministic methods, and this is still the industry practice in most regions. At the center of the power system operations challenge is the UC problem, which has received substantial attention among researchers and practitioners for a long time (e.g., Hobbs et al. 2001 [3], Padhy 2004 [4]). Much of the recent work on analyzing the impact of renewable resources on power system operations has also been done with deterministic models. Below, we provide a few examples of deterministic analysis of the impacts of wind power on the grid, as a background to the discussion of probabilistic approaches to operating reserves and stochastic UC to follow in the next chapters.

Modeling the impact of wind energy on power system operations is not a new exercise. For example, Bossanyi (1983) [19] uses a simulation model to study the integration of wind energy into the power system, and finds that most of the system cost savings achieved are due to the savings of fossil fuels, while in the long term additional savings result from re-optimization of the plant mix. Söder (1993) [20] analyzes the impact of wind power on the hydrothermal power system in Sweden, with a focus on how increasing wind penetration influences the need for operating reserves. Watson et al. (1994) [21] simulate the operation of the power system in England and Wales with different wind penetration levels, with a focus on the use of wind power forecasting. They demonstrate economic benefits of fossil fuel savings from wind energy and

emphasize the need for effective planning of online reserve capacity. Milligan (1996) [22] studies possible methods to incorporate wind power into production cost models for the power system. In particular, he compares a simplified load duration curve approach with a chronological UC/ED method and concludes that the latter is more suitable for analyzing grid impacts of wind power, since it captures the temporal variability in the wind resources and its consequences for scheduling and dispatch of other system resources.

The modeling of the impact of renewable energy on the power system has received increasing attention in recent years, as the penetration of wind and solar resources has increased rapidly in various regions of the world. Ummels et al. (2007) [23] propose a UC/ED simulation framework based on a deterministic dynamic programming approach method to analyze the impact of wind power on power system operations, reliability, and emissions. The model is applied to the Dutch power system for wind power penetrations up to 22% of load. The results show that wind power reduces operating costs and emissions. However, significant wind curtailment occurs at higher wind penetration levels. They conclude that the Dutch system has sufficient flexibility to handle wind power uncertainty and additional reserves are not required. Moreover, somewhat surprisingly, wind prediction has insignificant impact on system operation cost, emission, or wind curtailment.

Tuohy et al. (2007) [24] investigate the impact of rolling commitment to UC/ED decisions in a power system with wind power. They use both a simplified UC/ED formulation and the commercial software PLEXOS in a case study of the power system in Ireland, considering the impact of wind power on the need for operating reserves and also different levels of forecast accuracy. They find that by scheduling the system more often, the amount of extra reserve to be carried to compensate for wind uncertainty decreases, depending on the flexibility of the plants in the system. This reduces the costs of operating the system. There is a trade-off between reduced costs due to more frequent commitment, the ability of wind forecasts to be made more accurately, and the increased costs of more flexible plants. Delarue et al. (2008) [25] present an adaptive UC strategy to evaluate the value of forecasting in power system operations. The strategy is tested on a power system with a total installed capacity of 15,000 MW, focusing on the impact of the net load forecast. They find that with a shorter forecast horizon, the operating cost is higher even if a perfect forecast is used. Moreover, when a certain error is imposed on the forecasts, the deviations from the optimal solution and cost become larger. Bakirtzis et al. (2014) [26] propose a method to unify UC and ED with variable dispatch horizon, time resolution, and model complexity. The method is tested on the Greek power system. The results indicate that the proposed method can provide adequate capacity and ramping capability to follow abrupt variable generation changes within reasonable execution times.

The articles briefly outlined above represent examples of studies where deterministic scheduling and dispatch methods have been improved to account for the variability and uncertainty in renewable energy, e.g., through estimation of operating reserves that account for renewable resources, more frequent scheduling decisions, and finer time resolution modeling. The large-scale renewable energy integration studies in the United States, such as the Eastern Wind Integration and Transmission Study [27] and the Western Wind and Solar Integration Study [28] [29] investigate a large range of challenges related to the integration of wind and solar power into the grid. So far, these types of integration studies have also relied on deterministic

UC/ED models to simulate power system operations with increasing levels of renewable energy, albeit with increasing sophistication in terms of detailed input data, treatment of operating reserves, geographical areas covered, simulation periods and time resolution, etc.

The work presented by Restrepo and Galiana (2011) [30] is an example of a different direction. The authors present a hybrid deterministic/stochastic UC model where wind power uncertainty is accounted for through a probabilistic reserve constraint, accounting for the impact of wind power curtailment on residual demand uncertainty. A case study on the RTS-96 system illustrates that the proposed UC reduces operational costs, up- and down reserves, and the number of on/off operations. Probabilistic reserves and stochastic UC formulations represent two important research directions in the context of power system operations with renewable energy that are discussed in more detail in the next two chapters.

For more detailed discussions on general developments with regard to challenges and potential solutions for power system operations with increasing levels of wind and solar power, see Smith et al. 2007 [6], Xie et al. 2011 [7], Holttinen et al. 2013a [8], and Bessa et al. 2014 [9].

3 ADDRESSING UNCERTAINTY AND VARIABILITY WITH OPERATING RESERVES

In this chapter, we review probabilistic methods proposed in the literature to estimate the needs for operating reserve requirements. After giving a brief overview of the topic, we start the review by looking at methods proposed for traditional systems where thermal power plants make up the majority of the resources. We then review different proposed approaches to factoring in the impact of increasing levels of variable renewable resources in the determination of operating reserve requirements.

3.1 OVERVIEW

Operating reserves play a key role in maintaining reliability in the power system by handling uncertainty and variability in power system operations. Keeping an adequate level of operating reserves is important, as the system reliability deteriorates with too few reserves and the system cost increases if excessive reserves are carried. It is therefore not surprising that operating reserve requirements in power systems have been a topic of interest for a long time.

Estimating the need for operating reserves is a probabilistic problem, since it has to account for uncertain events, such as the potential outage of generators or transmission lines, load forecasting errors, and, more recently, errors in the prediction of renewable resources. Probabilistic methods were proposed at early stages to estimate the risk of supply shortage and to derive adequate operating reserve levels accordingly to meet certain reliability standards. Still, industry practice so far has to a large extent focused on deterministic heuristics, such as the N-1 rule, i.e., keep at least as much reserve in the system as is needed to make up for the single largest contingency. The rapid expansion of wind and solar power has increased the interest in probabilistic approaches to estimate operating-reserve requirements, as these resources may substantially add to the overall variability and uncertainty in the system. Hence, recent efforts have been devoted to characterizing wind and solar forecast uncertainty, as well as incorporating this information into probabilistic algorithms for estimating reserve requirements. One trend is to move from static rules to dynamic estimates of reserve requirements, which reflect the current and forecasted status of the system. Moreover, the set of reserve categories, which traditionally consist of a combination of regulation, spinning, and non-spinning reserves in United States electricity markets, is being revisited and new products introduced that to some extent are tailored to the specific characteristics of renewable resources (Figure 4). There are also recent developments in electricity markets in terms of the valuation and pricing of reserves, and how it relates to the energy market.

Regardless of how the operating reserves are derived, they are typically imposed as explicit constraints on deterministic formulations of the UC and ED problems. An alternative approach is to use stochastic UC/ED formulations to model the reserves implicitly by representing uncertainty within the problem formulation. The latter approach has theoretical advantages, but stochastic formulations give rise to computational challenges and open questions regarding pricing of energy and reserves under uncertainty, as discussed in the next chapter. In

contrast, advances in reserve estimation methods tend to impose a lower computational burden and can more easily be integrated into current operational and market practices.

3.2 PROBABILISTIC EVALUATION OF RESERVES IN THERMAL SYSTEMS

As early as 1963, Anstine et al. (1963) [31] proposed a methodology to estimate risk of not being able to meet demand, accounting for the load level, the available generation capacity, load forecasting errors, and the outage probabilities of the generators. The authors illustrate how the probabilistic algorithm, which is implemented on a computer, can be used to calculate the required level of spinning reserves to maintain a certain reliability level, e.g., a risk of failure of 1 in 1000, in the Pennsylvania-New Jersey-Maryland system. The authors point out that "No numerical calculations can alone establish the proper level of risk for a particular system. The selection of a satisfactory level of reliability requires the exercise of informed judgment." Garver (1966) [32] proposes a probabilistic method based on loss-of-load mathematics to estimate the load-carrying capability of generating units. The effective capability of a new unit corresponds to the load increase that the system may carry for a given fixed reliability level, measured in terms of the loss-of-load probability (LOLP). The method is applied to long-term reliability assessment and generation expansion planning. Many of the same principles introduced in these early papers are also applied in more recent work on probabilistic estimation of operating reserves and system reliability.

Dillon et al. (1978) [33] propose an iterative procedure to ensure that the UC solution meets a certain risk level. The authors point out that "considerable thought should be given to the actual risk level employed and its relationship to extra system cost." Gooi et al. (1999) [34] propose a probabilistic assessment of the need for spinning reserves to account for generator outages and load forecasting uncertainties in order to keep reliability within a specific risk level. The reserve assessment is built into a Lagrangian-based UC formulation through an iterative procedure. The paper shows that a probabilistic reserve assessment can be used in short-term generation scheduling to derive a spinning-reserve requirement that is appropriate for the individual failure rates of committed units and load forecasting errors, so that the probability of lost load can be controlled.

Chattopadhyay and Baldick (2002) [35] propose a UC formulation with a simplified statistical approximation to integrate the capacity outage probability distribution that enables the solution of the UC problem into one single optimization problem. Hence, there is no need for iterations to check and ensure the reliability of the candidate solutions, as was typically the case with earlier methods. The proposed algorithm considers outages of thermal generators and it is shown in a case study of the IEEE RTS system that it is a computationally efficient algorithm with reasonable accuracy. Moreover, the proposed probabilistic algorithm ensures a better risk profile than the deterministic single-contingency benchmark for determination of reserves. The choice of reserve strategy is also shown to have significant impacts on energy prices. Bouffard and Galiana (2004) [36] propose a hybrid deterministic-probabilistic approach to the reserve-constrained market-clearing algorithm for a single-period scheduling model and illustrate how the resulting scheduling, system reliability, cost, and prices depend on the selected reliability level. Moreover, the proposed algorithm ensures that system reliability stays within upper

bounds, as opposed to traditional deterministic approaches. The proposed algorithm has two advantages: one is that it behaves in a manner consistent with purely probabilistic criteria such as LOLP and Expected Load Not Served; second, its mathematical form is compatible with powerful mixed-integer linear programming tools.

Ortega-Vazquez and Kirschen (2007) [37] present a new approach to scheduling the optimal amount of spinning reserves in thermal systems. In this case, the need for operating reserves is not represented in terms of a constraint or target for reliability. Rather, the proposed algorithm accounts for the trade-off between the cost of providing energy and reserves and the expected cost of load shedding, by minimizing the sum of both. Optimal reserve levels are found for individual time periods and then imposed on the standard multi-period UC problem. A case study compares reserves, costs, and LOLP for the proposed approach and traditional N-1 method, illustrating large differences in results under some conditions. Another approach is taken by Wang et al. (2009) [38], who propose a UC and market-clearing formulation where system security is represented in terms of selected contingencies rather than reserve constraints. The formulation is solved using Benders' decomposition, solving for the contingency cases as a sub-problem. The method allows for LMPs for energy as well as spinning and non-spinning reserves that reflect the cost of security, i.e., the commitment and positioning of units to make sure all selected contingencies can be met. However, the likelihood of those contingencies is not considered, so it is not a probabilistic approach, strictly speaking. Finally, Ahmadi-Khatir et al. (2013) [39] propose an efficient algorithm for probabilistic spinning-reserve assessment in multi-control zone power systems, based on imposing reliability constraints on each system. Reliability metric calculations are extended for a multi-area power system while the power flow injections in the area borders are accounted for. The results indicate benefits in terms of both reduced costs and increased reliability compared to conventional methods, since energy and reserve resources are efficiently shared among the areas.

3.3 PROBABILISTIC EVALUATION OF RESERVES WITH RENEWABLE ENERGY

Consideration of the impact of variable renewable energy on the need for operating reserves is a more recent endeavor. In an early paper on this topic, Söder (1993) [20] proposes a probabilistic method to estimate the need for different types of operating reserves in the Nordic power system. The proposed method finds the required amounts of instantaneous, fast, and slow reserves that are needed to ensure that contingencies and forecasting errors can be addressed with a certain probability. Load and wind power distributions are both assumed to follow a Normal distribution. The method is applied in an analysis of the wind-thermal-hydro system in Sweden, where reserve requirements as well as resulting reserve margins are evaluated. More recently, Doherty and O'Malley (2005) [40] present a probabilistic method to determine the amounts of reserves that are needed to meet certain reliability targets over the year, accounting for wind and load forecasting errors (both assumed to follow Normal distributions) as well as thermal generator outages. The reserves are split into primary, secondary, and tertiary reserves, depending on the required response times. Results show that increasing wind capacity gives rises to higher reserve needs, primarily for slow-responding reserves. Reserve needs also increase with the forecast horizon.

Makarov et al. (2009) [41] propose a statistical algorithm to derive the required capacity, ramp rate, and duration of load following and regulation reserves in the California system. The approach is based on the so-called "swinging window" method, considering the detailed timeline and procedure for real-time scheduling and dispatch in the CAISO market. Load and wind power forecasting errors are represented as truncated normal distributions. Results show a significant increase in both load-following and regulation requirements with higher wind power penetration. The proposed method was first introduced by Makarov et al. (2008) [42], who applied it to the Bonneville Power Administration system. Ortega-Vazquez and Kirschen (2009) [43] expand on their earlier work [37] to consider the impact of wind power forecast errors, along with load forecast errors and thermal outages, on the optimal amount of spinning reserves. Forecast errors for load and wind are assumed to follow normal distributions. The optimal reserve level is found by minimizing the sum of operating costs and the expected cost of unserved load, using a heuristic similar to the one in [37]. In a case study, the proposed approach is compared to other reserve strategies, including the single-largest-contingency and the 3.5 sigma approaches. The proposed probabilistic approach leads to lower cost and also lower standard deviation in cost, but results are sensitive to assumptions about the value of lost load.

Matos and Bessa (2011) [44] propose a method to derive operating reserve requirements based on probabilistic wind power forecasts, rather than an assumed forecast error distribution. The method finds the optimal amount of operating reserves, factoring in wind power forecast uncertainty—represented as a non-parametric probabilistic forecast—as well as wind turbine outages, along with load forecast errors and thermal generator contingencies. The proposed approach uses value-functions to model the trade-offs between cost of reserves and the risk of unserved energy, under the assumption that the generation schedule has already been determined. A case study of the power system in Portugal compares operating-reserve levels with the proposed method for several benchmark strategies. Trade-offs between reserve cost and the risk of unserved energy are also illustrated. Zhou and Botterud (2014) [45] take a similar approach when they propose a dynamic operating-reserve demand curve (ORDC), also based on probabilistic wind power forecasts, along with load forecasting errors and generator outages. The proposed method enables a dynamic adjustment of operating reserves, based on the forecast uncertainty in the wind power generation. Rather than finding one target level of reserves, the demand curve reflects the marginal value of reserves to the system in terms of reduced likelihood of outages, as a function of how much reserve is being scheduled. The proposed ORDC can be used in day-ahead, intra-day, and real-time UC/ED and market clearing for co-optimization of energy and reserves. Results from a case study based on generation, load, and wind data from the state of Illinois indicate that the proposed ORDC has several advantages, including more stable energy and reserve prices, flexibility in dispatch, and possibly lower operating costs (in low-load periods), compared to benchmarks with price-inelastic reserve requirements.

3.4 OPERATING RESERVES IN INTEGRATION STUDIES AND INDUSTRY

Operating reserves have long been recognized to be a key challenge for integration of renewable energy. The role of reserves in power system balancing and the impact of wind power are discussed by Ackerman et al. (2007) [46], with a focus on industry practices in Europe. Ela et al. (2011) [47] discuss current trends in operating reserves and the impact of variable renewable

energy on the need for such services. Holttinen et al. (2013b) [48] present an overview of current and proposed operating reserve categories, methods to estimate the impact of wind power on reserves, and current developments in selected countries. A general trend is the recognition that the industry needs to move from traditional static reserve rules towards more dynamic reserve requirements. For instance, the large-scale wind and solar integration studies performed in the United States, such as the Eastern Wind Integration and Transmission Study [27] and the Western Solar and Wind Integration Study [28] [29], use dynamic reserves. The needs for regulation, spinning, and non-spinning reserves were determined as a function of the variability and uncertainty in the renewable resource, conditioned on certain explanatory variables, such as wind power output for wind and the clear sky index for solar power. The level of operating reserves carried in the system will ultimately influence the ability to balance the system, e.g., measured in terms of the Control Performance Standard 2 (CPS2) in the United States. In a recent study of the integration of solar PV, Mills et al. [49] investigate the relationship between operating reserves, CPS2 performance, and the cost of integrating solar PV into system operations. The inverse relationship between balancing performance and integration costs is illustrated.

Industry is also gaining interest in probabilistic reserve calculations, accounting for the impact of renewable energy. For instance, in Portugal and Spain, countries that both have substantial renewable-energy penetration levels, system operators have conducted studies comparing deterministic and probabilistic reserve methods [48]. In the United States, the system operator in Texas, Electric Reliability Council of Texas (ERCOT), uses a probabilistic assessment to determine the need for regulation and non-spinning reserves [50]. The assessment is done monthly and ensures specific confidence levels for the reserve capacity's ability to meet the variability and uncertainty in the net load (i.e., load minus wind power) based on historical data of forecast and realized load and wind power. Moreover, an ORDC was introduced into the ERCOT real-time market in 2014 [51]. The probabilistic demand curve also reflects historical forecasting errors, but is not dynamically adjusted depending on forecast uncertainty and is only used for pricing purposes.

A related direction is the potential introduction of alternative operating reserve products to address the uncertainty and variability in renewable resources. As an example, so-called flexi-ramp products have already been introduced in the CAISO and MISO markets to ensure that sufficient ramping capacity is available in real-time dispatch, accounting for the uncertainty in renewable energy and other resources and its impact on ramping requirements. Wang and Hobbs (2014) [52] investigate the introduction of such flexi-ramp constraints in deterministic ED and compare it to a stochastic formulation. The results show that adding a flexi-ramp requirement can enhance system flexibility and lower the cost of managing variability in net loads, bringing the solution closer to the stochastic ideal. However, the amounts of flexi-ramp acquired have large impacts on the results. Wang and Hobbs (2015) [53] expand on their initial study by introducing real-time UC decisions into the analysis. They arrive at similar results, i.e., flexi-ramp reserves may improve the performance compared to a basic deterministic formulation, but the performance still falls short of the stochastic ideal. Hence, the authors conclude that careful considerations are needed when considering the introduction of such markets. The interaction between the flexi-ramp reserve and other reserve products is an important issue that was not investigated in these two papers.

For a more detailed discussion on current industry practices and future developments in the United States, see Ela et al. (2011) [47], Holttinen et al. (2013b) [48], and Ela et al. (2014) [18].

3.5 STOCHASTIC SCHEDULING FORMULATIONS WITH IMPLICIT RESERVES

An alternative approach to imposing explicit operating reserve requirements in scheduling and dispatch is to model uncertainties implicitly in the underlying optimization problem. Bouffard et al. (2005a) [54] propose a stochastic market-clearing scheme with UC that considers potential contingencies of generators and transmission lines. Under this formulation, unlike the deterministic reserve-constrained UC, the reserve services are determined by economically penalizing the operation of the market by the Expected Load Not Served. The likelihood of contingencies is based on historical forced outage rates. Mathematically, it becomes a stochastic programming problem that can be solved with mixed-integer linear programming. The potential economic benefits of the stochastic market-clearing formulation, which considers the expected costs of reserve deployment and involuntary load shedding in the optimization problem, are illustrated in a corresponding case study by Bouffard et al. (2005b) [55]. Test results indicate lower expected costs of stochastic compared to deterministic market clearing. Market-clearing prices for energy and security are also lower with the stochastic approach. However, the authors acknowledge that computational feasibility will be a challenge on larger systems. The authors expand on their work on stochastic market clearing by introducing wind power as a stochastic element in the stochastic programming formulation (Bouffard and Galiana [2008] [56]). A case study illustrates that the proposed approach leads to lower expected operating costs and the ability to integrate more wind power than is the case with a deterministic market-clearing formulation that schedules reserves to handle all wind scenarios as contingencies.

Morales et al. (2009) [57] propose a stochastic program to schedule day-ahead energy and reserves, accounting for potential real-time realizations of wind power. Results show that overall system costs decrease as wind penetration increases, but the reserve cost attributable to wind power is significant. They conclude that modeling the stochastic behavior of wind resources is required to determine adequate reserve levels. Papavasilliou and Oren (2011) [58] also propose a two-stage stochastic UC model for determining reserve requirements in the presence of wind power forecast uncertainty, and a method for generating and weighing the wind scenarios that are used in the stochastic UC on the basis of their importance for the underlying decision problem. The proposed model is shown to outperform deterministic reserve schedules, e.g., those based on a certain percentage of forecast peak load and wind power generation, as well as other rules used in wind integration studies.

The papers discussed briefly in this section focus on finding optimal reserve levels under uncertainty in renewables and system contingencies and are, in effect, stochastic programming formulations of the UC problem. A wide body of literature has been devoted to this topic in recent years, motivated by the rapid increase in renewable generation across the globe. We discuss stochastic scheduling approaches in more detail in the next chapter.

3.6 SUMMARY

The different methods for operating reserves under uncertainty that are discussed in this chapter are briefly summarized in Table 2.

TABLE 2 Summary of Operating Reserve Approaches, Accounting for Uncertainty

Application	Method	Reference
Probabilistic reserve estimates for thermal systems	Probabilistic estimates of LOLP	Anstine et al. (1963) [31] Garver (1966) [32]
	Reserves in UC to meet reliability constraints	Dillon et al. (1978) [33] Gooi et al. (1999) [34] Chattopadhyay and Baldick (2002) [35] Bouffard and Galiana (2004) [36] Ahmadi-Khatir et al. (2013) [39]
	Reserves in UC to minimize total system cost, including expected load shedding	Ortega-Vazquez and Kirschen (2007) [37]
	Ensure that all selected contingencies can be met in UC	Wang et al. (2009) [38]
Probabilistic reserve estimates with renewable energy	Meet probabilistic reliability target assuming Normal distribution for renewable energy forecast error	Söder (1993) [20] Doherty and O'Malley (2005) [40] Makarov et al. (2008, 2009) [41] [42]
	Reserves in UC to minimize total system cost, including expected load shedding	Ortega-Vazquez et al. (2009) [43]
	Reserve evaluation based on probabilistic wind power forecasts	Matos and Bessa (2011) [44] Zhou and Botterud (2014) [45]
	Operating reserve demand curves	ERCOT (2013b) [51] Zhou and Botterud (2014) [45]
	New reserve products	Wang and Hobbs (2014, 2015) [52] [53]
	Reserves in integration studies and industry developments	Ackerman et al. (2007) [46] Ela et al. (2011) [47] Holttinen et al. (2013) [48] Enernex (2010) [27] GE Energy (2010) [28] Lew et al. (2013) [29] Mills et al. (2013) [49] ERCOT (2013a) [50] ERCOT (2013b) [51] Ellison et al. (2012) [59] Ela et al. (2014) [18]

TABLE 2 (Cont.)

Application	Method	Reference
Stochastic scheduling with implicit reserves	Stochastic programming formulations of the UC problem	Bouffard et al. (2005a,b) [54] [55] Bouffard et al. (2008) [56] Morales et al. (2009) [57] Papavasiliou et al. (2011) [58]

4 STOCHASTIC PROGRAMMING FORMULATIONS OF THE UNIT COMMITMENT PROBLEM

To better account for the increasing levels of uncertainty in power systems with renewable resources, UC formulations based on stochastic programming have received substantial attention in recent years. This chapter focuses on scenario-based stochastic UC formulations. The next chapter will cover alternative approaches for UC under uncertainty, including robust programming, chance-constrained programming, interval optimization, and fuzzy sets.

4.1 OVERVIEW

Deterministic mathematical programming models are applied to optimization problems with known parameters. However, for most real-world problems, there are some uncertain parameters. For example, the generation from wind and solar resources, as well as the load, are not known with certainty in the UC problem. If the probability distribution of the uncertain parameters can be estimated, potential realizations of those parameters can be sampled from the distributions. Then, the goal is to find some policy that is feasible for all potential realizations and optimize the expectation of some objective, which might be a function of decisions as well as the uncertain parameters. In general, stochastic programming is a framework to solve optimization problems with uncertain input parameters, which are typically represented by a set of scenarios, generated from the known or estimated probability distributions.

Traditional deterministic operational strategies may not be adequate to address the increased uncertainty from renewable resources and to balance the supply and demand in an economical and reliable way. For instance, deterministic scheduling based on the expected wind power output could be far from optimal if the realized wind power deviates far from its expectation. One potential approach is to introduce multiple scenarios to represent the potential realization of wind power within its distribution boundary, and develop a scenario-based UC formulation based on stochastic programming. In this case, the optimal scheduling policy minimizes the expected total cost for the system (or maximizes expected profit for an individual generation company).

Stochastic programming-based UC formulations attempt to represent the multi-stage nature of power system operations, in which uncertainties are unfolded or mitigated gradually over time at each stage. The most commonly applied formulation is the two-stage UC model, where the UC schedule for thermal units is determined in the first stage and the corresponding dispatch solution is determined in the second stage for each potential realization of the uncertain parameters, as estimated at the time of the first stage. The second-stage problem is also called the recourse problem. Stochastic programming was first introduced into the UC problem modeling with uncertainty from load [60] [61], and then to systems with uncertainties from electricity prices [62], and system contingencies [63]. The approach has frequently been applied in recent years, as renewables are integrated into the power system on a large scale. In most cases, the uncertainty in parameters is represented by a set of scenarios, which for instance could be

generated from an empirical distribution function and Monte Carlo simulations. The uncertainty representation may take the form of individual scenarios or scenario trees. The stochastic UC formulation typically applies all dispatch-related constraints to each scenario and optimizes the expectation of the objective value over all scenarios.

An advantage of the stochastic formulation is that it represents uncertainty explicitly within the problem formulation, as opposed to the use of operating-reserve constraints in deterministic formulations. The expected value objective is well aligned with general theory for rational decision-making under uncertainty. In theory, stochastic UC should therefore lead to improved scheduling decisions if the scenarios are generated from a probabilistic forecast that adequately captures the underlying probability distribution of the uncertain parameters. However, scenario generation for renewable resources is a challenging task. Stochastic formulations also involve a substantial increase in the computational burden of the UC problem. Moreover, there are challenges in terms of integrating a different scheduling approach into existing operational and market processes. The potential benefits of stochastic UC must be weighed against these challenges.

4.2 A GENERAL AND SPECIFIC STOCHASTIC UNIT COMMITMENT FORMULATION

An example of a general stochastic UC formulation is presented below:[1]

$$\min\{\mathrm{C}(\mathrm{u}) + \mathrm{E}_{\zeta\in\mathrm{H}}(\mathrm{F}(\mathrm{u}', \mathrm{g}, \zeta)|\mathrm{Au} \leq b_u, u \in U, W^{\zeta} g^{\zeta} \leq b_g^{\zeta}, \mathrm{u}' = \mathrm{u}, \forall\, \zeta \in \mathrm{H}\ \}$$

where u is the UC decision and $\mathrm{C}(\mathrm{u})$ is the cost associated with UC, e.g., start-up and/or shut down cost. H is the scenario set to represent the joint distribution of uncertainty. ζ is a scenario in the scenario set. $\mathrm{F}(\mathrm{u}', \mathrm{g}, \zeta)$ is the dispatch cost associated with the UC decision u' and dispatch decision g for scenario ζ. $\mathrm{E}_{\zeta\in\mathrm{H}}(\mathrm{F}(\mathrm{u}, \mathrm{g}, \zeta)$ is the expected costs associated with dispatch over the whole scenario set. The objective is to minimize the total cost for the ED schedule and the expected corresponding dispatch results over all scenarios. The constraints can be categorized into two groups. The first group ($\mathrm{Au} \leq b_u, u \in U$, where A and b are system parameters and U is the set of all feasible UC schedules) is related to unit physical constraints of the generators that may affect UC decisions, such as minimum on/off hours. The second group is related to ED constraints, which are scenario-dependent. Particularly, $\mathrm{u}' = \mathrm{u}$ is called non-anticipativity constraints, which means that the UC decision in the first stage is independent of the scenarios. W and/or b_g are input parameters with uncertainty, and could, for example, represent wind power supply or system contingencies. The formulation is considered as a two-stage stochastic problem, where the UC schedule is determined in the first stage on the basis of estimated scenarios of uncertainty that are unknown at that time, while the second stage corresponds to a recourse or dispatch cost given the UC schedule and a particular realization/scenario of the uncertainty.

[1] The model presented here is from the perspective of a system operator or a vertically integrated utility company minimizing system costs. The model for a generation company maximizing profits is similar.

More specifically, below we present a high-level UC formulation with uncertainty from renewable resources. The objective is to minimize the total of three types of costs. First, the total expected sum of fuel costs from thermal units, *FC*. Second, the expected costs of reserve not served, *C*(*RNS*), and energy not served *C*(*ENS*), taken over a set of renewable energy scenarios *s*. These two parts correspond to the expected dispatch cost in the general formulation. Third, the thermal units' start-up costs, *SC*, which are not scenario-dependent, correspond to the UC cost in the general formulation.

Objective function:

$$\text{Min} \sum_{s} prob_s \cdot \left\{ \sum_{t,i} \left[FC_{t,i}^{s} + C(RNS_t^s) + C(ENS_t^s) \right] \right\} + \sum_{t,i} SC_{t,i}$$

At a high level, the scenario-dependent constraints can be listed as follows:

Energy balance constraints:

$$\sum_{i} gen_{thermal,i,t}^{s} + gen_{renewable,t}^{s} = load_t - ENS_t^s, \ \ \forall\, t, s$$

Reserve balance constraints:

$$\sum_{i} r_{thermal,i,t}^{s} \geq (OR_{reg,t} + \ OR_{renewable,t}^{s}) - RNS_t^s, \ \ \forall\, t, s$$

where $gen_{thermal,i,t}^{s}$ is the generation from thermal unit i at time t with scenario s; $gen_{renewable,t}^{s}$ is the generation from renewable energy at time t with scenario s; $load_t$ is the load demand at time t; ENS_t^s is the amount of energy not served at time t with scenario s; $r_{thermal,i,t}^{s}$ is the amount of reserve provided by thermal unit i at time t with scenario s; $OR_{reg,t}$ is the amount of operating reserve required by traditional resources at time t; and $OR_{renewable,t}^{s}$ is the amount of explicit operating reserve required to account for uncertainty and variability from renewable energy at time t and scenario *s*. If the set of scenarios considered in the problem formulation covers the full range of uncertainty, $OR_{renewable,t}^{s}$ can be set to zero.

Operational constraints:

The operational constraints are similar to those in deterministic UC formulations, representing ramping limits, min up/down times, etc., for thermal generators.

4.3 REVIEW OF STOCHASTIC PROGRAMMING UNIT COMMITMENT FORMULATIONS

As illustrated by the general formulation, the natural extension from the traditional UC formulation is to apply dispatch-dependent decisions and constraints to all scenarios, and minimize the expected operating cost (or profit maximization, for generation companies); for example, see [64] [65] [66] [67] [68]. One of the related directions is to co-optimize the energy and reserve by including the cost of explicit reserves and penalty of reserve curtailment in the objective functions and/or including the reserve requirement balance in the constraints. Kalantari et al. (2013) [69] include the cost of the scheduled up-and-down reserve in the UC objective function. Operating reserves are closely related to system reliability, as discussed in the previous chapter. A second direction is to introduce risks associated with different events to model the case where deviations from scheduled dispatch may have an impact on system operations and costs [64] [70]. For example, Zhang et al. (2014) [70], consider the risks of loss of load, wind curtailment and branch overflow caused by wind power supply uncertainty and include the cost of these risks in the objective function. Wu et al. (2008) [71] model the spatial constraints of generation units and transmission lines, random component outages, and load forecast uncertainty, and calculate the optimal amount of reserve to balance operating cost and system reliability. In this study, the optimal operating point of the power system is based on the minimum total cost, which includes operating and Expected Energy Not Served (EENS) costs. This optimal point is determined by power system characteristics, generating unit and transmission constraints, fuel prices, and load-shedding costs. The optimal reserve level is implicitly determined by this optimal point, which indicates that the marginal cost of additional reserves at the optimal point is equal to the marginal cost of reducing EENS at that point. A third direction is to include emissions in the model, either in the objective function or in the constraints. For example, Wu et al. (2007) [63] consider the emission constraints from a generation company's perspective.

In stochastic programming-based UC, a set of scenarios is used to represent the distribution of uncertain input, such as wind generation. One approach to generate scenarios is to use Monte Carlo simulation to produce realizations of the random variables from empirical probability functions, usually derived from historical data. A second method is to construct a scenario tree [72] for multistage problems, where each path from the root to a leaf corresponds to a scenario. Each scenario is assigned a probability value, and the total equals one. The former approach is more common in two-stage formulations, where all uncertainty is assumed to be revealed between the two stages, while the scenario tree generation method can capture the correlations and evolution of uncertainty for multiple stages [73] [74]. Some applications may have a scenario selection/screening process to remove low-impact, low-probability scenarios [67] [75].

Theoretically, although stochastic programming methods provide a consistent framework for modeling rational decision-making processes in power system operation under uncertainty, when it comes to larger-scale problems they usually cannot produce the optimal solution within the allowed time frame. Rather, a feasible solution within a preset gap is usually found. Moreover, the discrete scenario set used to represent the distribution of uncertain parameters may not be sufficiently accurate to capture the full distribution. Taking all these factors into

consideration, it might be necessary to schedule a certain amount of explicit operating reserve to hedge against the unaccounted uncertainty seen by the stochastic UC formulation. Reserve can be either scheduled as an explicit system input parameter as in deterministic formulations, or scheduled implicitly within the stochastic formulation [65] [76] [77] [78]. Ruiz et al. (2009) [65] present a combined method with an implicit stochastic formulation and reserve strategy for the efficient management of uncertainty in the UC problem. Numerical studies show that UC solutions obtained for the combined approach are robust and superior in terms of production cost, cost variance, and reliability.

To take advantage of more accurate forecasts closer to the real-time dispatch, additional scheduling processes could be added to revise the UC schedule from previous stages for fast-response units. An example is the reliability assessment commitment processes that are present in most ISO/RTO markets in the United States. Correspondingly, multi-stage UC formulations have been applied to model this decision process [79] [80] [81] [82]. Tuohy et al. (2009) [81] shows that more frequent scheduling contributes to further improvements in system operations as a result of reductions in forecast uncertainty.

Stochastic dynamic programming (SDP) is an optimization method that can represent the full multi-stage nature of decision problems under uncertainty, by solving sub-problems recursively and finding optimal decision strategies across possible states. However, SDP formulations for larger systems quickly suffer from the “curse of dimensionality.” Hence, relatively few UC formulations based on SDP are proposed in the literature, owing to the high computational burden. Hargreaves and Hobbs (2012) [83] propose a SDP algorithm to model the UC and ED of a power system with wind uncertainty. It is observed that the SDP model has greater benefits for higher wind penetration levels, in terms of production cost and penalty for curtailment. Schneider et al. (2013) [84] model a power system with demand response and load shifting based on a stochastic UC formulation. The model is solved by a method combining approximate dynamic programming and progressive hedging, and tested on the CAISO system. The results indicate that potential savings in energy costs can be achieved by using demand-side resources, especially for growing shares of intermittent capacity in energy generation.

Uckun et al. (2015) [85] propose an intermediate multi-stage formulation, where the standard non-anticipative constraints that are typically imposed across all scenarios in two-stage formulations are relaxed and only applied within a set of time segments and for certain realizations of wind power. Hence, a more flexible scheduling strategy is obtained. Numerical results on 6-, 24-, and 118-bus systems indicate significant cost savings (1–2%) compared to the standard two-stage stochastic UC model. For the small test system, the proposed approach is shown to produce results that lie between the standard two-stage and a full multi-stage formulation.

Another potential way to improve the accountability of renewable variability and uncertainty is to set dispatch constraints to finer time resolution. Wang et al. (2013) [86] propose a stochastic UC formulation with sub-hourly dispatch constraints. The model is solved using an improved Benders’ decomposition algorithm. The model is tested on the IEEE 118-bus system and proven to outperform existing stochastic UC models with respect to production cost and load curtailment reduction.

To reduce the computational time and improve the tractability of the UC solution, some variations of the standard models have been proposed recently. Dvorkin et al. (2015) [87] propose a hybrid UC model, which applies stochastic programming and interval optimization in sequence to optimize the UC schedule. The model is tested on the IEEE 24-bus reliability test system. The results show that the proposed hybrid formulation can balance the robustness of the interval UC and the low expected cost of the stochastic UC. Specifically, the schedules produced by this hybrid formulation depend on the value of lost load (VOLL). When this value increases, the proposed model schedules more resources to reduce the uncertainty exposure. Wu and Shahidehpour (2014) [88] propose a constrained ordinal optimization-based security-constrained UC method to reduce the computational requirement while keeping a good UC schedule. The model is tested on the modified IEEE 118-bus system with various wind penetration levels. The results indicate that the proposed model could produce a better stochastic security-constrained UC solution, in terms of higher Monte Carlo simulation estimation accuracy and computational efficiency.

4.4 UNCERTAINTY MODELING

The uncertainties in power system operations are mainly from component outages, load forecasts, fuel prices, and renewable energy forecasts. For some systems which are closely connected to neighboring electricity markets, uncertainty in neighboring electricity market prices is another important consideration. Traditionally, system uncertainties are accounted for by scheduling additional generation capacity to provide operating reserves, as discussed in the previous chapter. Recently, stochastic optimization-based models have been developed to integrate uncertainties into the model formulation. There are also some models combining a stochastic optimization formulation and additional operating reserve to better hedge the risk from the full range of uncertainties. In all of these formulations, the uncertainties need to be modeled and represented in some ways that can be used by successive decision models.

When it comes to wind power integration, wind power forecasting has strong implications for the security and costs associated with decision-making in systems with high penetration of wind power. This includes power system scheduling and dispatch, and the quantification of operating reserve requirements associated with these decisions. Conventionally, an uncertain parameter is represented by its expected value and the methods to produce the expected value are referred to as point forecasting methods. However, when it comes to more complicated applications under uncertainty, the expected value representation cannot provide sufficient information. For example, stochastic programming formulations require a set of scenarios to represent the distribution of wind power supply. Moreover, in applications for wind power market participation and trading, an accurate representation of price forecasting as well as wind power forecasting uncertainty have an important function in controlling the trade-off between risk and return when wind energy is scheduled in the electricity market. Recently, several probabilistic forecasting methods have been developed to estimate the distribution of wind power supply, which can provide uncertainty information or even the full probability distribution functions. Along with stochastic optimization formulations, the uncertainty distribution can lead to more robust or economical decision support for the operation of power systems with wind and other renewable resources.

The wind power forecast is described by using random variables, which may be expressed in many forms: (1) probability mass function; (2) moments of distributions (e.g., mean, variance, skewness, kurtosis); (3) a set of quantiles and interval forecasts; and (4) probability density functions (pdfs) or cumulative distribution functions. The pdfs are generic and can be deduced to all of the other forms. The use of each uncertainty representation is case-dependent. For instance, when faced with a decision-making problem, if one uses a parametric representation of the uncertainty, then the moments of the distribution are sufficient to quantify the uncertainty. Specifically for scenario-based stochastic programming-based decision-making problems, it is important that the uncertainty representation allow for the generation of scenarios to represent the probability distribution in a discretized form. Probabilistic forecasting, scenario generation, and scenario reduction are discussed in more detail below.

4.4.1 Probabilistic Forecasting

The problem consists of forecasting the wind power pdf at time step t for each look-ahead time step $t+k$ of a given time horizon, knowing a set of explanatory variables (e.g., point forecasting and measured wind power output). Mathematically, this can be formulated as follows:

$$f_p\left(p_{t+k} \middle| X = x_{t+k|t}\right) = \frac{f_{P,X}(p_{t+k}, x_{t+k|t})}{f_X(x_{t+k|t})} \ ,$$

where p_{t+k} is the variable of the forecasted wind power for look-ahead time $t+k$, $x_{t+k|t}$ are the explanatory variables forecasted for look-ahead time step $t+k$ given available information at time step t, $f_{P,X}(p_{t+k}, x_{t+k|t})$ is the joint density function, $f_X(x_{t+k|t})$ is the marginal density of X, and f_p is the conditional density function of wind power at time step $t+k$.

In general, the probabilistic forecasting models can be categorized into three groups on the basis of the forecasting model inputs[2]: (1) Approaches based on numerical weather prediction (NWP) point forecasts [89] [90] [91] [92] [93] [94]; (2) approaches based on power output point forecasts [95] [96] [97] [98] [99] [100] [101] [102] [103] [104] [105] [106]; and (3) approaches based on NWP ensembles [107] [108] [109] [110].

The performance of the probabilistic forecasts can be evaluated by three metrics: calibration (or reliability), sharpness, and skill score. Calibration is a measure of how well the forecast quantiles match the observed values. For instance, the wind generation should be below the 5% quantile only 5% of the time. If the realized wind power generation is below the 5% forecast more frequently, there is a positive bias in the forecast, measured in terms of deviation from perfect calibration. The sharpness metric represents the tendency of the probabilistic forecast towards discrete forecasts, measured by the mean size of the forecast intervals (i.e., the distance between quantiles). Hence, sharpness is a measure of the width of the forecast distribution. The skill score is intended to provide global information about the performance of a

[2] For details of each different category, see [112].

model in a single measure to complement the evaluation functions of sharpness and calibration. It should be well designed to make sure the perfect forecasting models receive the best scores. For more information about the performance metrics, see [111] [112].

4.4.2 Scenario Generation

The forecasted pdfs express the probability distribution of the wind power forecast for a specific point in time. However, for UC problems, it is important to take inter-temporal relationships in the forecast uncertainty into account. Moreover, it is difficult to directly use a continuous distribution function in a stochastic programming formulation. For these reasons, scenarios are a more appropriate representation of the uncertainty.

A commonly used method is to generate a set of scenarios with a Monte Carlo simulation from an assumed or estimated distribution with correlation matrix.

Pinson et al. (2009) [113] propose a statistical method to generate scenarios of short-term wind generation that accounts for both the interdependence structure of prediction errors and the predictive distributions of wind power production. The method is based on the conversion of a set of random variables composing probabilistic forecast series into a multivariate Gaussian random variable. The temporal interdependence structure is represented by the covariance matrix, which is recursively estimated with new incoming information. The method is tested on a multi-MW wind farm for a period of 2 years.

Pinson et al. (2012) [114] argue that existing evaluation frameworks do not work well because they only focus on the marginal predictive densities and do not allow one to discriminate among scenarios. The paper presents multivariate verification tools, as well as diagnostic approaches based on event-based verification. The tools are tested with data from France and proven to be valuable discrimination tools for various scenario trajectories.

Morales et al. (2010) [115] propose a methodology to characterize the stochastic processes pertaining to wind speed at different geographical locations via scenarios. The methodology is accurate in reproducing historical wind-speed series, as well as computationally efficient. The generated wind-speed scenarios retain the main statistical properties, including the marginal distribution, temporal correlations and spatial correlations.

To better accommodate the non-anticipativity constraints in any multi-stage stochastic formulation, scenario tree construction methods have been developed to approximate the distribution with a finite number of scenarios by minimizing the pertinent error while retaining accuracy in specified statistical information.

Lowery et al. (2012) [116] present a scenario tree tool to alter forecast error statistics to facilitate the study of how these statistics impact on UC and system operations. The tool, along with the WILMAR [117] tool, is tested on Ireland’s power system in 2020. The results show that variance, skewness, and kurtosis all have impacts on system operation, while variance has the

most impact. Moreover, representation of variance, skewness, and kurtosis can affect the dependency of commitment upon flexible units.

4.4.3 Scenario Reduction

A large number of scenarios may make the associated stochastic programming problem computationally intractable. The purpose of scenario reduction techniques is to reduce the number of scenarios while keeping most of the stochastic information. In general, a subset of the original scenario set is selected on the basis of certain criteria, and new probabilities are assigned to the preserved scenarios.

Growe-Kuska et al. (2003) [118] propose scenario reduction and scenario tree construction algorithms for power management problems. The algorithms are tested on a German utility system. The results show that the scenario reduction algorithms can reduce the computational requirement, while retaining similar accuracy in reduced after the number of reduced scenarios reaches a certain level.

Morales et al. (2009) [119] propose a forward selection-based scenario reduction algorithm for future market trading problems. The algorithm measures the distance of scenarios by the distance of optimal values of the corresponding decision problem if the scenarios are input. The results show that the proposed algorithm produces improved profitability results, in terms of both magnitude and variability, while keeping a smaller number of scenarios.

Sumaili et al. (2011) [120] propose a clustering-based scenario reduction method. Scenarios are clustered into a finite number of groups on the basis of the distances, and a representative scenario is either selected or generated from each group and assigned a probability. The validity of the reduction method is validated in a simplified UC problem.

Feng et al. (2014) [121] propose a heuristic scenario reduction method based on forward selection, which selects scenarios on the basis of their cost and reliability impacts. The model is tested on data from ISO New England. The results show that the proposed method provides more reliable commitment schedules with similar costs for either a single-day or a rolling-horizon decision procedure.

4.5 MODELING TOOLS AND SOLUTION ALGORITHMS

The intensive computational requirement to solve stochastic programming-based UC models is a barrier to practical applications. The computational complexity increases exponentially when the scale of the system increases, and the problem becomes more challenging when new system features are represented in the model. For example, re-scheduling of fast generation units will add one more stage or introduce binary commitment variables in the second stage. The number of scenarios has a great impact on solution times as well.

When the scale of the system is modest, some conventional integer programming methods and associated solvers can still work. For example, CPLEX is widely used, e.g., in [69] [70] [76] [77] [87] [122]. CBC is used in [123]. FICO Xpress is used in [74].

For larger-scale applications, decomposition-based solution algorithms have been developed to tackle the computational challenges by decomposing the original problem, for instance into one master problem with a set of subproblems that are solved separately, either in parallel or sequentially. The decomposition methods for stochastic UC are usually categorized into three groups on the basis of how the full problem is broken down to construct subproblems: primal methods with subproblems assigned to UC decision variables (single-unit subproblem), dual methods with subproblems assigned to scenarios (single-scenario subproblem), and methods that break down the original problem into several single-stage subproblems for the multi-stage UC problem. In practice, Lagrangian relaxation or augmented Lagrangian relaxation is applied to move some of the coupled constraints into the objective functions. The major decomposition methods include Benders' decomposition and progressive hedging.

The decomposition method is first introduced to the UC problem in [60] [61] [124]. In these studies, Lagrangian relaxation is first applied to nonanticipativity constraints. Then, in [60], progressive hedging is applied to decompose the problem into a set of single-stage subproblems. In [124] and [61], Benders' decomposition is also applied to break the original problem into a set of single-scenario subproblems. Recently, these approaches have been widely applied with several variations, as in [67] [71] [73] [84] [86] [125] [126] [127] [128] [129] [130] [131] and so on.

Some additional solution methods are reported as well. Pappala et al. (2009) [132] propose a stochastic formulation for the UC-ED problem and a heuristic solution technology based on an adaptive particle swarm optimization. It is claimed that, with the proposed formulation and solution technology, one can obtain more economical schedules with similar system reliability.

4.6 PERFORMANCE COMPARISON METRICS

To evaluate the performance of the stochastic programming methods on UC problems relative to conventional deterministic programming methods, several metrics are proposed from computational, economic, reliability, and environmental perspectives.

Economic metrics include expected cost savings and related scheduling results such as committed conventional generation capacity, committed operating reserve, renewable energy utilization, number of start-ups from thermal units, and so on. For example, [60] [62] [65] [66] [67] [126] [127] [133] compare the expected cost from stochastic and deterministic UC models. The savings range from 0.4% to 2.7%, depending on the specific simulation settings such as wind penetration levels, optimality gap, and operating reserve policy. [65] [67] report that the amounts of committed conventional generation capacity and operating reserve decrease with a stochastic programming UC model. [79] [80] report that there are fewer start-ups for base units, and more for middle-merit gas units, while the total is slightly higher with stochastic UC. [65]

reports that less operating reserve is required with stochastic UC formulation. [86] proposes a stochastic UC formulation with sub-hourly dispatch constraints and reports that the proposed model commits more units than traditional models to account for sub-hourly wind power variability and achieves more production cost savings and less load curtailment.

Reliability metrics include the amount of unserved energy and reserve. [65] reports that both the amounts of curtailed operating reserve and load are reduced with the stochastic model. [67] [79] [80] report the same observation on load curtailment.

Overall, there is no standard approach for evaluation of the results from stochastic UC models. At the same time, a number of assumptions go into these types of analyses, making it somewhat challenging to compare results across different studies.

4.7 TEST BEDS

The most commonly used test systems are various standard IEEE and other test systems, including the IEEE Reliability Test System and its variations [65] [66] [69] [70] [76] [87], the IEEE 118-bus system [63] [71] [86] [88] [125] [127] [130], and the WECC-240 system [134]. Some reduced real-world system are also tested, including a reduce CAISO system [67] [84], the CAISO interconnection with the WECC [126] [135], the Eastern Interconnection [82] [136], nationwide power systems [74] [80] [81] [83] [137] [138], and some real-world utility company systems [60] [61] [62] [77] [122] [124] [128] [129] [133].

Most of the studies model the two-stage power system scheduling problem with UC as the first stage and ED as the second, also corresponding to the so-called two-settlement electricity market-clearing procedure. Some papers investigate multi-stage scheduling on a rolling basis to reschedule fast units when more recent updates on uncertainty are revealed [79] [81] [82].

4.8 REAL-WORLD SYSTEM CASE STUDIES AND INDUSTRIAL APPLICATIONS

As discussed in the previous section, some case studies use real-world systems as test beds. Such studies are discussed in more detail below. For example, Ela et al. (2010) [82] analyze the benefit of stochastic UC and rolling updating strategies under uncertainty by applying the WILMAR model to the U.S. Eastern Interconnection system. The results show that stochastic planning and rolling updates in the scheduling process have great benefits. The rolling updates show more benefits overall than the use of stochastic UC. Meibom et al. (2007) [137] present the use of WILMAR to optimize the UC considering the uncertainties from wind power generation and apply the model to the power systems of Denmark, Germany, Finland, Norway, and Sweden. It is concluded that, with higher wind power penetration, the total operation costs of the system decrease and the value of saved water in hydro storage increases. The avoided costs per additional unit of wind power production decreases. [79] [80] [81] compare the performance of UC schedules produced by stochastic and deterministic models on future scenarios of the Irish power system with various levels of wind power penetration by using WILMAR as well. It is

observed that the schedule from the stochastic model is less costly and can provide more reliable system operation. Sturt and Strbac [74] present an efficient formulation of the stochastic UC problem for a possible future British power system in 2030 by using a quantile-based scenario tree structure to avoid the need for explicit operating-reserve requirements. This structure is proved to provide statistically significant cost improvements over several existing models. Hargreaves and Hobbs [83] apply a SDP model to the Netherlands system. Abrell and Kunz (2015) [139] develop a stochastic electricity market model to analyze the impact of uncertain wind generation on one week's operation of the German electricity system.

Industry is also showing interest in stochastic UC formulations. As the major real-world users of UC models, some of the ISOs and utility companies in the United States have started some pilot studies to test the application of stochastic UC methods to their systems. In [140], MISO identifies stochastic programming as one potential solution to address system operation under increasing uncertainty. The solution is expected to reduce reserve margins, which in turn will reduce the production cost. In [141], ISO New England applies the stochastic programming formulation and the branch-and-cut solution method to its 24-hour UC problem with 309 conventional units and the aggregated wind generation for the New England area from April to September 2006. The results show that the expected cost from the stochastic programming formulation is as much as 20% lower than that from the deterministic formulation with a 5% wind penetration level, at the cost of longer computational time. Moreover, teamed with Sandia National Laboratory [142] [143], ISO New England applied the stochastic programming formulation and progressive hedging solution method to test the Eastern Interconnection system, and solved the problem by using a supercomputer.

4.9 SUMMARY

Finally, we summarize the discussion in this section through a set of tables. Table 3 summarizes formulations that are used for the stochastic UC problem with uncertainty. Table 4 summarizes the solution methods that are used to solve the UC problem. Table 5 summarizes the methods of probabilistic wind power forecasting.

Table 6 summarizes reported cost savings compared to that of deterministic formulations.

TABLE 3 Summary of Stochastic UC Formulations

Application	Specification	References
Extension from traditional UC formulation	Minimizing expected operating cost	Papavasiliou and Oren (2013a) [67]
	Combination of stochastic UC and operating reserve constraints	Kalantari et al. (2013) [69] Ruiz et al. (2008) [76], (2009a) [65], (2009b) [77], (2010) [66] Zhou et al. (2013) [78]
	Risk indices associated with different events	Li et al. (2007) [64] Zhang et al. (2014) [70] Wu et al. (2008) [71]
	Emissions	Wu et al. (2007) [63]
Multi-stage decision-making	Rolling-basis decision-making	Meibom et al. (2008) [79] Tuohy et al. (2008) [80], (2009) [81], Ela et al. (2010) [82]
	Relaxed non-anticipativity constraints between time segments	Uckun et al. (2015) [85]
	Stochastic dynamic programming	Hargreaves and Hobbs (2012) [83] Schneider et al. (2013) [84]
Hybrid formulations	Stochastic optimization and interval optimization combined	Dvorkin et al. (2015) [87]
	Stochastic optimization and robust optimization	Zhao and Guan (2013) [144]
Uncertainty representation	Scenario tree/bundles	Li et al. (2007) [64] Takriti et al. (1996) [60], (2000) [62] Wu et al. (2007) [63], (2008) [71], Shiina and Birge (2004) [73] Nowak et al. (2000) [129] Meibom et al. (2007) [137], (2010) [79], (2011) [136] Pappala et al. (2009) [132] Tuohy et al. (2008) [80] Ela et al. (2010) [82] Sturt and Strbac (2011) [74]
	Scenario sets	Ruiz et al. (2008) [76], (2009a) [65], (2009b) [77], (2010) [66] Papavasiliou and Oren (2013b) [126] Nowak et al. (2005) [128] Carøe et al. (1998) [61], (1997) [124] Wang et al. (2008) [127] Huang et al. (2014) [130] Wu and Shahidehpour (2014) [88] Kalantari et al. (2013) [69] Dvorkin et al. (2015) [87] Wang et al. (2013) [86] Constantinescu et al. (2011) [123]
	Scenario screening/selection	Papavasiliou and Oren (2013a) [67] Feng et al. (2013) [75]

TABLE 4 Solution Methods for Stochastic UC Problems

Method	Specification	References
Commercial solver	Cplex	Kalantari et al. (2013) [69] Zhang et al. (2014) [70] Ruiz et al. (2008) [76], (2009) [77] Dvorkin et al. (2015) [87] Zhao et al. (2013) [122] Abrell et al. (2015) [139]
	CBC	Constantinescu et al. (2011) [123]
	FICO Xpress	Sturt and Strbac (2012) [74]
Decomposition-based	By scenarios	Carøe et al. (1998) [61], (1997) [124] Wu et al. (2008) [71], (2012) [125] Papavasiliou and Oren (2013a) [67], (2013b) [126] Wang et al. (2013) [86] Wang et al. (2008) [127] Nowak et al. (2005) [128] Goez et al. (2008) [131]
	By generation unit	Shiina and Birge (2004) [73] Nowak and Römisch (2000) [129] Huang et al. (2014) [130]
	By stage/time period	Takriti et al. (1996) [60] Schneider et al. (2013) [84]
Others	Particle swarm optimization	Pappala et al. (2009) [132]

TABLE 5 Summary of Probabilistic Wind Power Forecasting Methods

Input data	References
NWP point forecast-based	Bremnes (2004) [89], (2006) [91] Lange (2005) [90] Jeon and Taylor (2012) [92] Kou et al. (2013) [93] Messner et al. (2014) [94]
Power output point forecast-based	Nielsen (2006) [95] Pinson (2006) [96], (2012) [102] Bludszuweit (2008) [97] Juban et al. (2007) [98] Møller et al. (2008) [99] Carpinone et al. (2010) [100] Bessa et al. (2012) [101] Sideratos and Hatziargyriou (2012) [103] Haque et al. (2014) [104] Wan et al. (2014) [105] Li et al. (2015) [106]
NWP ensemble-based	Nielsen et al. (2006) [107], (2007) [108], Pinson and Madsen (2009) [109] Möller et al. (2013) [110]

TABLE 6 Summary of Cost Savings of Stochastic UC Methods Compared to Deterministic Benchmark.

Metric	Performance	References
Operating cost saving	1.6% when using stochastic program	Gröwe et al. (1995) [133]
	0.73% for case of generation shortage; 0.39% to 1.18% for case of uncertain load	Takriti et al. (1996) [60]
	Approximately 4% over the expected-value policy (in deterministic formulation)	Takriti et al. (2000) [62]
	1.26%% savings on operational cost	Wang et al. (2008) [127]
	0.6% compared to that of the deterministic case	Tuohy et al. (2008) [80]
	Between 2.8% and 3.8%	Pappala et al. (2009) [132]
	Between 0.82% and 1.22% (optimal expected cost) with respect to the traditional policy.	Ruiz et al. (2009) [65]
	Between 1% and 1.8%	Ruiz et al. (2010) [66]
	Cost is only 1% higher than the model with perfect wind power information	Constantinescu et al. (2011) [123]
	0.6% cost saving compared to best deterministic strategy	Wang et al. (2011) [68]
	1.7% cost saving compared to best deterministic case	Zhou et al. (2013) [78]
	1.93% to 2.77% daily relative to best deterministic strategy, depending on wind penetration level	Papavasiliou and Oren (2013a) [67]
	Stochastic UC outperforms security-constrained UC by 5.4% relative to the average daily cost	Papavasiliou and Oren, (2013b [126])
	Cost increase of 0.04%	Abrell, et al. (2015) [139]

5 OTHER UNIT COMMITMENT FORMULATIONS UNDER UNCERTAINTY

In this chapter, we review a few alternative formulations of the UC problem under uncertainty from renewable resources, including formulations based on chance-constrained programming, robust programming, and interval optimization.

5.1 OVERVIEW

In a power system with uncertain generation resources, it might be allowable for constraints that include the random variables to be violated for some perturbations of those variables. One of the general optimization approaches under uncertainty is to apply a defined probability level for the violation of a set of constraints. This is called chance-constrained programming. The mathematical formulation with chance constraints can be reformulated into a deterministic model and solved by conventional solution methodologies. A potential advantage of this approach is that specific probabilistic reliability levels can be ensured through the chance-constrained formulation. However, like in stochastic programming, this method relies on the assumption that underlying probability distributions of uncertain variables can be accurately estimated.

Instead of modeling uncertainty in a probabilistic sense as in stochastic programs or chance-constrained formulations, robust optimization models uncertainty using a deterministic set, e.g., as a set of possible scenarios or a range of possible values for the uncertain parameters. Hence, knowledge of the uncertainty set's probability distribution is not required. One of the advantages of the robust optimization approach is that it provides a robust solution that is immune to any possible outcome of the uncertainty set, which is an important aspect in the security-constrained scheduling and planning of electric power systems. The robust optimization approach often solves the so-called minimax problem, which minimizes the worst-case cost that is maximized over the uncertainty set. The robust programming method is conservative by definition, since it focuses on minimizing the cost in the worst case scenario [145]. It can be argued that this is not a rational objective function since it does not align with axioms for rational decision making under uncertainty. However, the conservativeness of the robust solution can be controlled by the construction of uncertainty sets and a budget of uncertainty.

Other miscellaneous UC formulations include interval optimization and fuzzy set theory, which are also discussed in this chapter. These two approaches are both non-probabilistic. In interval optimization, uncertain parameters are modeled as intervals. The optimization is done for a central forecast while ensuring that the solution is feasible for all transitions within the full uncertainty set. Interval programming is therefore less conservative than robust programming, which minimizes the cost in the worst-case scenario. With interval optimization models, an uncertainty range for the objective function can also be estimated, which obviously depends on the uncertainty in input parameters. Fuzzy set theory is also an approach where decision problems are formulated as mathematical programming models with imprecise parameters, i.e., in terms of inexact constraints and fuzzy objective functions in the optimization problem. In fuzzy set theory, uncertainty is represented with membership functions, which describe the

possibility that an event will happen. In fact, interval numbers are a special instance of a fuzzy set, i.e., an interval number is a fuzzy set with a crisp membership function.

5.2 CHANCE-CONSTRAINED OPTIMIZATION BASED UNIT COMMITMENT

Chance-constrained programming was first introduced to model the UC problem with wind power generation in [146] [147]. Wind power generation is modeled as a multivariate random variable with mean and standard deviation for different time periods. The objective is to satisfy the net load (load – wind) with a specific probability level while minimizing the operating cost. The original problem is decomposed to a sequence of deterministic versions of the UC problem that converge to the solution of the chance-constrained program.

Wang et al. (2012) [148] formulate the UC problem as a chance-constrained two-stage stochastic program. A combined sample average approximation (SAA) algorithm is developed to solve the model. Chance constraints are applied to describe policies to ensure the utilization of wind power output. The approach is reported to be able to provide a solution that converges to the optimal one as the number of samples increases. The model is tested on a six-bus system and a revised 118-bus system. The computational results indicate that increasing the utilization of wind power output might increase the total power generation cost.

Pozo et al. (2013) [149] present a chance-constrained formulation with an alpha-quantile measure to determine the confidence level of meeting demand under K simultaneous contingencies, also factoring in load and demand uncertainty. CVar and duality theory are used to transform the chance-constrained optimization problem to a mixed-integer linear programming problem.

5.3 ROBUST OPTIMIZATION BASED UNIT COMMITMENT

Robust optimization methods have recently been introduced to power system UC and ED problems [144] [150] [151] [152] [153] [154] [155] [156] [157] [158] [159] [160] [161] [162] [163] [164] [165] [166].

5.3.1 Diverse Robust Unit Commitment Formulations

Zhang and Guan (2009) [150] are, to our knowledge, the first authors to apply robust optimization to the UC problem. The approach models the demand uncertainty in the real-time market as an uncertainty set with preset lower and upper bounds, but their model does not consider ramping and transmission constraints. Jiang et al. (2012) [152] develop an approach that includes applying robust optimization concepts and incorporating pumped-storage hydro units to accommodate wind power output uncertainty. It uses an uncertainty set that is obtained from historical data and defined as an interval. The uncertainty set includes the worst-case scenario and protects this scenario under the minimal increment of costs. The problem is formulated as a

two-stage minimax problem and solved using the Benders' decomposition algorithm. The results yield reduced operation cost and more reliable scheduling.

Uncertainty in power system operations involves not only renewable forecasting, but also forced outages. Street et al. (2011) [151], Xiong and Jirutitijaroen (2012) [153], and Wang et al. (2013) [157] present robust optimization approaches for the contingency-constrained UC problem with security criteria. Specifically, in [148], the n-K contingency-constrained UC problem is modeled as a worst-case bi-level programming problem where contingency states are modeled as decision variables and the parameters are allowed to vary to represent the generation unit availability under the contingency states. The model allows system operators to schedule power and reserves while explicitly considering all combinations of up to K generation unit outages.

Zhao et al. (2013) [156] present a three-stage robust UC model that has UC decisions in the first stage and dispatch decisions in the second stage, and then has uncertain demand response after dispatch decisions. The proposed approach can accommodate both wind power and demand response uncertainties. The wind power output varies within a given interval, and demand response can help accommodate wind power output uncertainty by lowering the unit load cost. The model is tested on the IEEE 118-bus system. Zhao and Guan (2013) [144] present a formulation that combines stochastic and robust UC models by introducing weights for the components for the stochastic and robust parts in the objective function. The weight can be adjusted on the basis of system operators' preferences.

Moreira et al. (2014) [162] present a nonparametric approach based on adjustable robust optimization to consider correlated nodal demand uncertainty in a joint energy and reserve scheduling model with security constraints. The uncertainty set is constructed considering model nonparametric correlations between nodal demands, owing to the observation that reserve costs are significantly influenced by correlation and conservativeness parameters.

An and Zeng (2015) [163] extend the modeling capacity of two-stage robust optimization and present two new robust UC variants: the expanded robust UC and the risk-constrained robust UC model. The model can accommodate multiple uncertainty sets. Liu et al. (2015) [167] present a stochastic robust framework for two-stage power system optimization problems with uncertainty. The model optimizes the probabilistic expectation of different worst-case scenarios with different uncertainty sets.

Liu and Tomsovic (2015) [166] propose a robust UC model considering uncertain price elasticity of demand. The results show that the average LMPs as well as the price volatility can be reduced.

Lorca and Sun (2014) [164] present an adaptive multi-period robust ED model and dynamic uncertainty sets for power system ED under high penetration levels of wind resources. Dynamic uncertainty sets explicitly model the relationship between uncertainties across decision stages and capture the temporal and spatial correlations of wind power output from multiple wind farms. The results demonstrate the benefits in terms of cost saving and reliability improvement. Lorca et al. (2014) [165] propose a multi-stage robust UC model which considers

non-anticipativity constraints in the dispatch process. This paper also proposes a constraint generation-based solution framework with various algorithmic improvements.

Minimax regret is another criterion that is being used in the robust optimization literature. Under the minimax regret criterion, as used by Jiang et al. (2013) [168], one minimizes the worst-case regret rather than the worst-case cost. The regret is essentially the difference between the resulting cost of a decision and the best achievable cost for a given scenario.

5.3.2 Solution Methods and Uncertainty Sets

Zhao and Zeng (2012) [154] develop a robust optimization model to minimize the system operating cost considering wind uncertainty and demand response uncertainty. Wind uncertainty is captured by a polytopic uncertainty set. The problem is solved by a column and constraint generation scheme. The results show that the proposed method is faster than Benders' cuts.

Bertsimas et al. (2013) [155] present a two-stage adaptive robust UC model considering uncertain nodal net injections. The nodal net injection uncertainty set models the variable resources, real-time demand variation, and interchange uncertainty. It develops a solution methodology based on a combination of Benders' decomposition and the outer approximation technique. The models proposed by Zhao and Guan (2013) [144] and Moreira et al. (2014) [162] are also solved by using Benders' decomposition. The results show that the framework can provide more robust and computationally tractable schedules.

Lee et al. (2014) [160] present a new model to dynamically incorporate critical transmission line constraints and their dual variables into a robust UC problem so that the solution time can be reduced by 40%–90%. The proposed heuristic column generation method, utilizing the information from the master problem, is found to be very effective, as it can generate a near-optimal solution for the robust UC problem with full transmission line constraints in approximately 15%–55% of the benchmark solution time

Xiong and Jirutitijaroen (2014) [161] propose a two-stage robust UC formulation to solve the UC problem with wind power generation uncertainty. The linear decision rule technique is applied to approximate the recourse decisions to make the solution computationally tractable.

Guan and Wang (2014) [158] present a different method to construct uncertainty sets based on historical data for robust UC problems addressing load, renewable energy generation, and demand response uncertainties.

5.3.3 Test Bed

The model proposed by Zhang and Guan (2009) [150] is tested on a 30-unit system and the results show that the cost and iteration number increase as the uncertainty set becomes larger. Street et al. (2011) [151] use a 100-unit test system without network constraints. An and Zeng (2015) [163] use 11 gas units without network constraints.

Most of the work on robust UC adopts the IEEE 118-bus system [149][152][156][157][160][164][165] and the IEEE Reliability Test System [153] [161] [162] [166] as test beds.

Bertsimas et al. (2013) [155] perform tests on the real-world large-scale system operated by ISO New England, and the computational results demonstrate the economic and operational advantages of the model over the traditional deterministic approach. Chen et al. (2014) [159] introduce a framework of robust optimization for the MISO look-ahead commitment routine. A case study of the MISO system shows that the robust programming approaches are promising and yet pose challenges that must be overcome to make these approaches practical for real-world applications. Lorca et al. (2014) [165] also perform tests on a real-world 2718-bus system.

5.4 INTERVAL OPTIMIZATION BASED UNIT COMMITMENT

Another approach that is gaining increasing interest in the research literature is the application of so-called interval optimization to the UC problem. This approach is based on describing uncertainty through interval arithmetic, i.e., the use of intervals, without any assumption about the probability distribution, to represent the range of potential outcomes for an uncertain variable. An early application of interval arithmetic to the power systems domain is reported by Wang and Alvarado (1992) [169], who propose a power flow algorithm based on interval arithmetic. UC formulations based on interval optimization find the minimum cost for the base realization of uncertain parameters (e.g., wind power) and ensure that the solution is feasible for the full interval representation of uncertainty. Hence, the approach is less conservative than robust optimization, which minimizes the cost under the worst-case outcome.

Recently, interval optimization has been used in several papers on power system operations with renewable energy. Wang et al. (2011) [170] formulate a UC problem where nodal net load (i.e., load minus wind power) is modeled as interval numbers. They identify a reduced scenario set that guarantees feasibility over all possible scenarios within the uncertainty intervals. In a case study, they demonstrate that the proposed interval UC outperforms a benchmark deterministic UC model in terms of both security and cost.

Wu et al. (2012) [125] compare interval UC with stochastic UC under wind power uncertainty. The proposed interval UC model provides a confidence interval for total operating costs with pessimistic and optimistic values along with the cost for the base case. In contrast, the stochastic UC model provides the expected operating cost over a reduced scenario set. Results on 6-bus and IEEE 118-bus test systems indicate that the stochastic formulation provides more stable solutions, but the computational burden is high. In contrast, interval optimization-based UC is computationally fast, but the estimated confidence intervals of costs are very sensitive to the assumed uncertainty intervals.

Zhou et al. (2014) [171] also compare interval UC with stochastic UC. They propose an extension to the standard interval UC formulation, which provides an estimate of the expected cost over the uncertainty interval. The expected cost is estimated using the so-called point estimate method, which calculates the expected cost from a specific sample set of the uncertain wind power input data. Case study results on the 6-bus and IEEE 118-bus systems indicate that

the proposed method can guarantee operational security within the interval uncertainty range and provide a good estimate of the expected costs. At the same time, the increase in computational speed compared to the standard interval optimization approach is modest.

Hu et al. (2014) [172] propose an interval UC model that considers correlation between wind and load variability. The model is solved using Benders' decomposition. The authors compare the proposed method with a robust UC model. Case study results on the IEEE 118-bus system show that the interval UC approach provides a solution with lower operating cost, while still maintaining operational feasibility within the interval uncertainty set. This is because interval UC minimizes the cost of the base forecast scenario as opposed to the worst-case scenario, which is the case for robust UC. The authors argue that optimizing for the base scenario is better aligned with current procedures for system operators in electricity markets. Finally, the representation of correlation in wind and load uncertainties eliminates some very unlikely scenarios, further reducing the conservativeness of the solution and contributing to improved economic efficiency.

Dvorkin et al. (2015) [87] propose a hybrid UC model, which applies stochastic programming and interval optimization in sequence to optimize the UC schedule. More specifically, stochastic programming is applied for the first part of the optimization horizon and interval UC for the second part, with the switching time optimized to minimize the expected costs. The model is tested on the IEEE 24-bus reliability test system. The results show that the proposed hybrid formulation can balance the robustness of the interval UC and the lower expected cost of the stochastic UC. Moreover, the schedules produced by the proposed hybrid formulation depend on the assumed VOLL. When this value increases, the proposed model schedules more resources than the stochastic UC to reduce the uncertainty exposure.

Pandzic et al. (2015) [173] expand on the work of Dvorkin et al. (2015) [87] and propose an improved interval UC formulation with wind power uncertainty, where the feasibility constraints are relaxed so that they do not cover the extreme transitions between upper and lower bounds of the wind power uncertainty interval, but rather are a function of ramps observed in a realistic wind power scenario set. Hence, the proposed approach leads to a less conservative UC strategy compared to the standard interval UC formulation. In a case study of the IEEE RTS-96 system, they compare the proposed method to the standard interval UC method as well as to stochastic and robust UC. They find that stochastic UC performs the best in terms of lowest expected cost. However, the improved interval UC outperforms the other strategies (i.e., standard interval UC and robust UC) and also has a 50% faster computational time compared to stochastic UC.

Liu et al. (2015) [174] consider the coordination of wind and hydropower and propose a price-based UC model where uncertainties in electricity prices (day-ahead and intra-day) as well as wind power are represented as intervals. Intervals for profits from using different operational strategies are estimated accordingly. A novel feature of the proposed model is that the risk preference of the decision-maker is represented through preference ordering of the resulting profit intervals. In a case study, the authors show that coordination of wind and hydro resources can reduce the uncertainty in profits. The results also show that the proposed interval method has a much lower computational burden than stochastic programming. The authors emphasize that

the interval methods do not require knowledge of the probability distribution of uncertain variables, but also recognize that it may be challenging to estimate uncertainty intervals, and that this has a large impact on results.

5.5 FUZZY SET BASED UNIT COMMITMENT

UC formulations are also proposed where uncertainty in renewable energy is addressed using fuzzy sets [175] [176] [177]. Fuzzy sets are used to describe the possibility that an event will take place, i.e., the assumption is that the probability distribution of the event is unknown, but so-called membership functions can be used to describe the possibility based on subjective assessments or expert judgments. In fact, the interval representation of uncertainty discussed in the section above is one specific instance of a fuzzy set, i.e., with crisp limits for the membership function. An early application of fuzzy set theory to power systems is reported by Miranda and Matos (1989) [178]. More recently, fuzzy sets have also been applied to power system operations with renewables, as briefly outlined below.

Hosseini et al. (2007) [175] present a UC formulation considering both integration and emissions requirements. The reliability and emission constraints are modeled as fuzzy constraints. The model is solved by using simulated annealing.

Venkatesh et al. (2008) [176] present two approaches for addressing wind power forecast uncertainty in day-ahead UC. The first approach uses a fuzzy objective function that considers EENS and total operating costs, whereas a second benchmark approach increases operating reserves in a deterministic UC.

Zhang et al. (2015) [177] present a UC model where demand response and electric vehicles are represented as two technologies that can help accommodate wind power variability and uncertainty in power system operations. The UC model is formulated as a fuzzy chance-constrained problem that is solved with particle swarm optimization. Wind power is considered in the operating reserve requirement, which is modeled as a chance constraint. A case study illustrates significant benefits of demand response and electric vehicles for grid operations with renewable energy.

5.6 SUMMARY

The different methods for UC formulation under uncertainty that are discussed in this chapter are briefly summarized in Table 7.

TABLE 7 Summary of Other Unit Commitment Formulations Under Uncertainty

Application	Specification	References
Chance-constrained programming	First introduced wind into chanced-constrained UC model	Ozturk (2003) [146] Ozturk et al. (2004) [147]
	Combined SAA algorithm	Wang et al. (2012) [148]
	Alpha-quantile measure to determine the confidence level	Pozo et al. (2013) [149]
Robust optimization	First introduced robust optimization into UC model with wind power	Zhang and Guan (2009) [150] Jiang et al. (2013) [152]
	Include n-K contingency in uncertainty set	Street et al. (2011) [151] Xiong and Jirutitijaroen (2012) [153] Wang et al. (2013) [157]
	Multi-stage robust UC	Zhao et al. (2013) [152]
	Combined stochastic and robust UC formulation	Zhao and Guan (2013) [141]
	Consider nonparametric correlations between nodal demands	Moreira et al. (2014) [158]
	Stochastic robust model, accommodating multiple uncertainty sets	An and Zeng (2015) [159] Liu et al. (2015) [163]
	Robust UC with demand response	Zhao et al. (2013) [156] Liu and Tomsovic (2015) [162]
	Considering non-anticipativity constraints in the dispatch process	Lorca and Sun (2014) [164] Lorca et al. (2014) [165]
	Minimax regret model	Jiang et al. (2013) [168]
	Column and constraint generation	Zhao and Zeng (2012) [154] An and Zeng (2015) [159] Lee et al. (2014) [160] Liu et al. (2015) [163]
	Benders' decomposition and outer approximation	Bertsimas et al. (2013) [155] Zhao and Guan (2013) [144] Moreira et al. (2014) [162] Zhao et al. (2013) [152]
	Dynamically include critical transmission line constraints	Lee et al. (2014) [160]
	Different method to construct uncertainty sets	Guan and Wang (2014) [154]

TABLE 7 (Cont.)

Application	Specification	References
Interval optimization	Wind power or net load uncertainties as interval numbers	Wang et al. (2011) [170] Wu et al. (2012) [125] Zhou et al. (2014) [171] Hu et al. (2014) [172]
	Hybrid stochastic/interval UC	Dvorkin et al. (2015) [87] Pandzic et al. (2015) [173]
	Interval UC with wind scenario ramps Interval opt. w/risk preferences	Liu et al. 2015 [174]
Fuzzy sets	Fuzzy set models with wind power uncertainty	Hosseini et al. (2007) [165] Venkatesh et al. (2008) [176] Zhang et al. (2015) [177]

6 CONCLUSIONS

In this report, we have reviewed current and potential applications of stochastic methods to the operations of power systems and electricity markets with increasing shares of renewable energy. The main findings of our review and potential directions for future work are summarized below.

The main findings from the literature review in this report can be summarized as follows:

- Probabilistic algorithms have been proposed and applied for a long time to estimate the need for operating reserves, accounting for the stochastic nature of transmission and generation outages. Still, deterministic operating rules based on heuristics (e.g., the single largest contingency rule for reserves) are often applied in practice.

- In recent years, there has been a surge in research on the application of stochastic methods for power system operations with high penetration of renewable energy. The stochastic UC problem has received most of the attention among researchers, but new approaches for probabilistic estimates of operating reserves have also been proposed.

- Although the majority of research has focused on stochastic programming formulations of the UC problem, alternative UC formulations under uncertainty, such as robust, chance-constrained, and interval programming are also gaining popularity.

- The main focus in the literature on stochastic scheduling and dispatch is on cost savings and reliability impacts, including metrics such as expected operating cost, committed thermal unit capacity, scheduled operating reserve capacity, renewable energy utilization, number of thermal unit startups, CO2 emissions, and load curtailment. However, there is also increasing attention to market implications, such as pricing of energy and reserves.

- Most of the studies report operating cost savings from using stochastic formulations, especially if potential savings from cost penalties for load or reserve curtailment are considered. There are also reports indicating that scenario-based stochastic UC formulations may lead to more frequent start-ups of thermal units. More specifically, while there are fewer start-ups of base load units, more start-ups occur for middle-merit natural-gas-fired units. The total number of start-ups is slightly higher to account for the variability of wind power supply.

- There are no standard frameworks or metrics to compare the pros and cons of different operational strategies. Except for using some standard IEEE test power systems for evaluation purposes (e.g., the IEEE 118-bus test system), the formulations proposed in the current literature have many different features (for example, demand response, storage, and emission constraints), making it difficult to compare the reported performance for operational strategies across different studies. Moreover, the reference or benchmark strategy often varies in different papers. In some studies, deterministic

operating strategies are used as a benchmark, while in some other studies alternative stochastic formulations are used as a benchmark.

- Industry adoption of probabilistic methods for operational decisions is still limited, but there is increasing interest in the topic. So far, in the United States, studies have been reported by ISO New England and MISO to test the applications of stochastic methods for operational decision-making on the large-scale systems they operate.

Directions for future work include the following:

- More systematic testing and comparison of different operational strategies, accounting for a larger set of the real-world issues, constraints, and potential future regulatory policies in power system and electricity market operations.

- A closer investigation of the interaction between explicit operating reserve requirements imposed by traditional reserve constraints and the implicit reserves provided by stochastic scheduling and dispatch formulations.

- Further investigation of the potential implications for pricing and market incentives under stochastic UC and ED, with the goal of providing efficient signals for operations and investments for all market participants.

- Further refinements of methods for probabilistic forecasting, scenario generation and reduction, as critical inputs to stochastic methods for power system operations.

- Further investigation of interaction between stochastic short term operations and risk-constrained long-term planning decisions.

- Development of stochastic methods for mid-term operation and coordination, such as maintenance scheduling from the system operator's point of view, as well as fuel and emissions planning from generation companies' perspectives.

- Testing on real-world and large-scale systems, with engagement from utility companies and system operators to validate the performance of stochastic methods and provide better quantitative estimates of benefits. Industry feedback and suggestions for improvements in research-grade algorithms are critical to developing the industrial tools needed for more economical and reliable power system operations with large shares of renewable energy.

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Energy Systems Division
9700 South Cass Avenue, Bldg. 362
Argonne, IL 60439-4854

www.anl.gov

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