Renormalization of Mass Flow Data in FCDPs for Thermal Efficiency Variation

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FUEL CYCLE OPTIONS CAMPAIGN

RENORMALIZATION OF MASS FLOW DATA IN FCDPs FOR THERMAL EFFICIENCY VARIATION

1. INTRODUCTION

The 40 Fuel Cycle Data Packages (FCDPs) [1] to be used for the 2013 Evaluation and Screening (E&S) of Fuel Cycle Options have been developed using assumptions [2] for the thermal efficiencies of the reactors in the different stages of the representative fuel cycle options for the 40 Evaluation Groups (EGs). Due to considerations about the potential for these different thermal efficiencies to bias the E&S results and maybe misinform readers, it has been requested to re-normalize mass flow data using the uniform thermal efficiency values for two additional sensitivity study cases. These cases are:

1. Renormalization using thermal efficiency value of 33% for all the reactors in a representative option.
2. Renormalization using thermal power (GWt) rather than electricity generation (GWe) for all the reactors in a representative option (corresponding to artificial weighting of the fuel cycle data using a turbine thermal efficiency of 100%).

It is noted that the thermal efficiency of any given External Driven System (EDS) has been adjusted separately for the transmuter (e.g., sub-critical blanket) and the overall system because a significant fraction of the power generated is used to support the system components, e.g., the accelerator in an EDS. The effective thermal efficiency values for the externally driven systems are provided in Table 1.

This memo documents the approach and results for the renormalization of the mass flow data contained in FCDPs.

2. APPROACH

Because physics considerations provide the basis for determining the power sharing to achieve the mass balance between different stages of the fuel cycle options, it is evident that in going from the data in the existing FCDPs to those for each of the two sensitivity study cases above, a common renormalization factor can be applied to the Mass Flow Data table of each of the FCDPs to get the re-normalized masses. To provide assurance about this principle, formulas have been derived to demonstrate that a single renormalization factor is required. Appendix A is the derivation of the formulas required to change the power sharing values and the renormalization factor value for the mass flow data tables. These formulas were validated by using the calculation approach for a representative option. The general forms of the formulas are

$$F_k^n = \frac{\omega_k}{\sum_i \omega_i F_i^o} F_k^o, \tag{1}$$

$$M_k^n = \frac{1}{\sum_i \omega_i F_i^o} M_k^o, \tag{2}$$

where the superscripts of *"n"* and *"o"* indicate the new and original thermal efficiencies, respectively, and the subscript denotes the stage number, and

F_k^n = Power-sharing fraction of k-th stage with new thermal efficiency,
F_k^o = Power-sharing fraction of k-th stage with original thermal efficiency,
M_k^n = Mass data of k-th stage in Mass Flow Data table of FCDP with new thermal efficiency,
M_k^o = Mass data of k-th stage in Mass Flow Data table of FCDP with original thermal efficiency,
ω_k = New to original thermal efficiency ratio of stage k (= η_k^n / η_k^o).

3. Results

These equations have been applied to the FCDPs for the 40 Evaluation Groups and the results for the new power sharing and renormalization values are summarized in Table 2. The table includes the

- EG group name (e.g., EG01/OT01A)
- Columns for pertinent data from the FCDPs (power sharing and thermal efficiency values)
- Columns for the uniform thermal efficiency of 33% (power sharing and renormalization values)
- Columns for thermal power generation weighting (power sharing and renormalization).

4. Application of the Results

The information provided in Table 2 can be used in the following way to derive the mass flow data for the two variations:

- The new power-sharing fractions for a given thermal efficiency variation are obtained directly from Table 2.
- To re-normalize the mass flow data for either a thermal efficiency of 33% or thermal power (GWt), use the renormalization factor given in Table 2 to multiple the mass flow data information entries contained in the Mass Flow Data table of FCDP file for the representative option of an Evaluation Group.

5. References

1. T. K. Kim, E. Hoffman, T. A. Taiwo, "Completion of Fuel Cycle Data Package System Datasheets for 2013 Evaluation and Screening," Argonne National Laboratory, ANL-FCT-333, FCRD-FCO-2013-000165, May 30, 2013.
2. "Assumption for Nuclear Energy Systems Analyses, Revision 3," FCRD-FCO-2012-000026, January15, 2013.

Table 1. Effective Thermal Efficiency Values of External Driven Systems

Option number	Design parameter	FCDP Data	Renormalization using thermal efficiency of 33%	Renormalization using thermal power [b)]
EG06	Stage	1		
	Thermal power, MWt	2441.0		
	Required electricity, MWe [a)]	195.0		
	Thermal efficient, %	44.4	33.0	100.0
	Effective thermal efficiency, %	36.41	25.01	92.01
EG07	Stage	1		
	Thermal power, MWt	1000.0		
	Required electricity, MWe	123.0		
	Thermal efficient, %	40.0	33.0	100.0
	Effective thermal efficiency, %	27.70	20.70	87.70
EG08	Stage	1		
	Thermal power, MWt	2000.0		
	Required electricity, MWe	195.0		
	Thermal efficient, %	43.0	33.0	100.0
	Effective thermal efficiency, %	33.25	23.25	90.25
EG16	Stage	2		
	Thermal power, MWt	840.0		
	Required electricity, MWe	84.9		
	Thermal efficient, %	40.0	33.0	100.0
	Effective thermal efficiency, %	29.89	22.89	89.89
EG33	Stage	1		
	Thermal power, MWt	840.0		
	Required electricity, MWe	46.0		
	Thermal efficient, %	40.0	33.0	100.0
	Effective thermal efficiency, %	34.52	27.52	94.52
EG34	Stage	1		
	Thermal power, MWt	840.0		
	Required electricity, MWe	43.68		
	Thermal efficient, %	40.0	33.0	100.0
	Effective thermal efficiency, %	34.80	27.80	94.80
EG35	Stage	2		
	Thermal power, MWt	840.0		
	Required electricity, MWe	75.0		
	Thermal efficient, %	40.0	33.0	100.0
	Effective thermal efficiency, %	31.07	24.07	91.07
EG36	Stage	2		
	Thermal power, MWt	840.0		
	Required electricity, MWe	42.0		
	Thermal efficient, %	40.0	33.0	100.0
	Effective thermal efficiency, %	35.00	28.00	95.00

a) Required electricity to support components of an External Driven System (e.g., accelerator).
b) Artificial thermal efficiency value.

Table 1. Effective Thermal Efficiency Values of External Driven Systems (Continued)

Option number	Design parameter	FCDP Data	Renormalization using thermal efficiency of 33%	Renormalization using thermal power
EG39	Stage	3		
	Thermal power, MWt	840.0		
	Required electricity, MWe	120.0		
	Thermal efficient, %	40.0	33.0	100.0
	Effective thermal efficiency, %	25.71	18.71	85.71
EG40	Stage	1		
	Thermal power, MWt	611.25		
	Required electricity, MWe	100.0		
	Thermal efficient, %	40.0	33.0	100.0
	Effective thermal efficiency, %	23.64	16.64	83.64

Table 2. Power-sharing Fractions and Mass Renormalization Factor for Thermal Efficiency of 33% and Thermal Power (GWt) for 40 Evaluation Groups

Thermal efficiency variation	Thermal efficiencies in FCDP System Datasheet						Renormalization using thermal efficiency of 33%				Renormalization using thermal power			
	Original thermal efficiency, %			Power sharing fraction , %			Power sharing fraction, %			Mass renormalization factor	Power sharing fraction, %			Mass renormalization factor
Stage	1	2	3	1	2	3	1	2	3		1	2	3	
EG01/OT01A	33			100.0			100.0			1.000	100.0			0.330
EG02/OT01B	50			100.0			100.0			1.515	100.0			0.500
EG03/OT01C	33			100.0			100.0			1.000	100.0			0.330
EG04/OT02	40			100.0			100.0			1.212	100.0			0.400
EG05/OT03	50			100.0			100.0			1.515	100.0			0.500
EG06/OT04	36.4			100.0			100.0			1.456	100.0			0.396
EG07/OT05	27.7			100.0			100.0			1.338	100.0			0.316
EG08/OT06	33.25			100.0			100.0			1.430	100.0			0.368
EG09/SL01	40			100.0			100.0			1.212	100.0			0.400
EG10/SL02	44.4			100.0			100.0			1.345	100.0			0.444
EG11/SL03	40			100.0			100.0			1.212	100.0			0.400
EG12/ML01	33	33		76.1	23.9		76.1	23.9		1.000	76.1	23.9		0.330
EG13/ML02	33.3	33.3		90.2	9.8		90.2	9.8		1.009	90.2	9.8		0.333
EG14/ML03	40	33		70.6	29.4		66.5	33.5		1.141	66.5	33.5		0.377
EG15/ML04	33	40		88.1	11.9		90.0	10.0		1.021	90.0	10.0		0.337
EG16/ML05	33	29.9		92.6	7.4		94.2	5.8		1.018	92.6	7.4		0.330
EG17/ML06	33	33		90.5	9.5		90.5	9.5		1.000	90.5	9.5		0.330
EG18/ML07	33	33		68.7	31.3		68.7	31.3		1.000	68.7	31.3		0.330
EG19/SC01	33			100.0			100.0			1.000	100.0			0.330
EG20/SC02	33			100.0			100.0			1.000	100.0			0.330
EG21/SC03	33.3			100.0			100.0			1.009	100.0			0.333
EG22/SC04	33.3			100.0			100.0			1.009	100.0			0.333
EG23/SC05	40			100.0			100.0			1.212	100.0			0.400
EG24/SC06	40			100.0			100.0			1.212	100.0			0.400
EG25/SC07	33			100.0			100.0			1.000	100.0			0.330

Table 2. Power-sharing Fractions and Mass Renormalization Factor for Thermal Efficiency of 33% and Thermal Power (GWt) for 40 Evaluation Groups (Continued)

Thermal efficiency variation	Thermal efficiencies in FCDP System Datasheet						Renormalization using thermal efficiency of 33%				Renormalization using thermal power			
	Original thermal efficiency			Power sharing fraction, %			Power sharing fraction, %			Mass renormal ization factor	Power sharing fraction, %			Mass renormal ization factor
Stage	1	2	3	1	2	3	1	2	3		1	2	3	
EG26/SC08	44.4			100.0			100.0			1.345	100.0			0.444
EG27/SC09	40			100.0			100.0			1.212	100.0			0.400
EG28/SC10	40			100.0			100.0			1.212	100.0			0.400
EG29/MC01	40	33		61.1	38.9		56.4	43.6		1.120	56.4	43.6		0.370
EG30/MC02	40	33		87.0	13.0		84.7	15.3		1.180	84.7	15.3		0.389
EG31/MC03	33	40		68.2	31.8		72.2	27.8		1.059	72.2	27.8		0.349
EG32/MC04	33	40		63.4	36.6		67.8	32.2		1.068	67.8	32.2		0.353
EG33/MC05	34.5	33		83.7	16.3		80.4	19.6		1.204	82.3	17.7		0.359
EG34/MC06	34.8	33		80.0	20.0		76.2	23.8		1.192	78.2	21.8		0.359
EG35/MC07	33	31.1		84.7	15.3		87.7	12.3		1.036	85.1	14.9		0.332
EG36/MC08	33.3	35		93.5	6.5		94.7	5.3		1.022	94.1	5.9		0.335
EG37/MC09	33.3	40.0	32.4	11.9	50.1	38	12.8	45.0	42.2	1.088	12.8	45.0	42.2	0.359
EG38/MC10	40	33		85.5	14.5		83.0	17.0		1.176	83.0	17.0		0.388
EG39/MC11	33	33	26	69.6	24.3	6.02	70.8	24.8	4.4	1.017	69.3	24.2	6.5	0.328
EG40/MC12	23.6	33		20.5	79.5		15.4	84.6		1.064	23.2	76.8		0.319

Appendix A: Derivation of Renormalization Formulas for Thermal Efficiency Variation

In this Appendix, the formulas of the power-sharing fraction of each stage and the common renormalization factor from the variation of the thermal efficiency are derived from the mass balance relationship in a typical two-stage fuel cycle option, but the derived formulas can be generalized to any number of stages in a fuel cycle option.

Transforming the Power Sharing

The relationship between the mass flow data per unit ***thermal power*** generation and ***electricity*** energy generation is

$$\hat{\dot{M}} = \eta \dot{M}, \tag{1}$$

where

η = Thermal efficiency of nuclear system,

$\hat{\dot{M}}$ = Mass flow data per unit thermal power generation, t/GWt-yr,

$\dot{M}$ = Mass flow data per unit electricity energy generation, t/GWe-yr.

In a typical two-stage fuel cycle option, there is material exchange between the first and second stage systems: e.g., the first stage produces extra materials and the second stage consumes them. At the equilibrium state, the net production rate in the first stage should balance the net consumption rate in the second stage;

$$F_1^o \times \dot{M}_1^o = F_2^o \times \dot{M}_2^o, \tag{2}$$

$$F_1^o + F_2^o = 1, \tag{3}$$

where, the superscript "o" indicates the values with the ***original*** thermal efficiency and

F_1^o = Fraction of power generation from the first stage.

F_2^o = Fraction of power generation from the second stage.

$\dot{M}_1^o$ = Net production mass per unit electricity generation from the first stage (t/GWe-yr).

$\dot{M}_2^o$ = Net consumption mass per unit electricity generation in the second stage (t/GWe-yr).

Similarly, the net production mass in the first stage should balance the net consumption mass in the second stage for the ***new*** thermal efficiencies,

$$F_1^n \times \dot{M}_1^n = F_2^n \times \dot{M}_2^n \tag{4}$$

$$F_1^n + F_2^n = 1 \tag{5}$$

where the superscript *"n"* indicates the values with the ***new*** thermal efficiency. In Eq (1), the mass flow data per unit thermal power generation (i.e., the value of left hand side) does not vary per thermal efficiency, while the mass flow data per unit electricity energy generation varies. Based on Eq (1), one can show the following relationships between new and original mass flow data,

$$\dot{M}_1^n = \frac{\eta_1^o}{\eta_1^n} \cdot \dot{M}_1^o \tag{6}$$

$$\dot{M}_2^n = \frac{\eta_2^o}{\eta_2^n} \cdot \dot{M}_2^o \tag{7}$$

Using Eqs (6), (7) and (2), Eq (4) becomes

$$\frac{F_2^o}{F_1^o} \times \frac{\eta_1^o}{\eta_1^n} F_1^n = \frac{\eta_1^o}{\eta_1^n} F_2^n . \tag{8}$$

Using Eqs (5) and (8), one can derive

$$\frac{\eta_1^o}{\eta_1^n} \frac{F_2^o}{F_1^o} = \left(\frac{\eta_2^o}{\eta_2^n} + \frac{F_2^o}{F_1^o} \times \frac{\eta_1^o}{\eta_1^n} \right) F_2^n . \tag{9}$$

Finally, one can derive the new fractions of electricity generation from the each stage,

$$F_1^n = \frac{\dfrac{\eta_1^n}{\eta_1^o}}{\left(\dfrac{\eta_1^n}{\eta_1^o} F_1^o + \dfrac{\eta_2^n}{\eta_2^o} F_2^o \right)} F_1^o , \tag{10.a}$$

$$F_2^n = \frac{\dfrac{\eta_2^n}{\eta_2^o}}{\left(\dfrac{\eta_1^n}{\eta_1^o} F_1^o + \dfrac{\eta_2^n}{\eta_2^o} F_2^o \right)} F_2^o . \tag{10.b}$$

Determination of a Common Renormalization Factor

The mass flow values in the "Mass Flow Data" table in each FCDP are calculated based on the following equations for the original and new thermal efficiencies,

$$M_{i,k}^o = C \cdot \dot{M}_{i,k}^o \times F_i^o , \tag{11}$$

$$M_{i,k}^n = C \cdot \dot{M}_{i,k}^n \times F_i^n , \tag{12}$$

where C is the normalization constant value to generate a certain amount of electricity. For the current FCDP template, the C value was defined as 100 GWe-yr. The subscripts of i and k denote the stage number and material kind (such as U, Pu, NU, etc.), respectively. By inserting Eqs (10), (6), and (7) into Eq (12), one can derive the relationships between the mass flow values for the new and original thermal efficiencies,

$$M_{1,k}^n = \frac{1}{\left(\dfrac{\eta_1^n}{\eta_1^o} F_1^o + \dfrac{\eta_2^n}{\eta_2^o} F_2^o \right)} M_{1,k}^o , \tag{13.a}$$

$$M_{2,k}^n = \frac{1}{\left(\dfrac{\eta_1^n}{\eta_1^o} F_1^o + \dfrac{\eta_2^n}{\eta_2^o} F_2^o \right)} M_{2,k}^o \tag{13.b}$$

So far, the power-sharing fraction formula of Eq (10) and the mass renormalization formula of Eq (13) have been derived for a typical two-stage fuel cycle option. ***In conclusion, note that Eqs (13.a) and (13.b) indicate that the mass renormalization factor is the same across the FCDP Mass Flow Data table regardless of the stage and material kind.***

Generalized Form of Formulas

The formulas of Eqs (10) and (13) could be generalized to any number of stage fuel cycle options such as

- Power-sharing fraction;

$$F_k^n = \frac{\omega_k}{\sum_i \omega_i F_i^o} F_k^o, \tag{14}$$

- Mass renormalization factor;

$$M_k^n = \frac{1}{\sum_i \omega_i F_i^o} M_k^o, \tag{15}$$

where the subscript denotes the stage number, and

ω_k = New to original thermal efficiency ratio of stage k (= η_k^n / η_k^o).

Validation of Formulas

In order to validate the derived formulas, the power sharing fractions and mass flow data for the thermal power generation case were calculated to obtain the mass balances between the stages in the fuel cycle option and the results were compared to those obtained with the formulas of Eqs (14) and (15). This exercise was performed for the Evaluation Group of 37, which has the most complicated fuel cycle option among the current 40 representative fuel cycle options.

The representative fuel cycle option for EG37 consists of PWR(LEU) to SFR(TRU/U,Th) to PWR(U-233/U). In this fuel cycle option, the TRU generated in all stages is recycled in stage 2 and the recovered uranium (mostly U-233) from the thorium blanket of stage 2 is used as the primary fissile material for stage 3. For this fuel cycle option, the loss rates during fuel fabrication and separations of 0.2% and 1.0% were assumed.

Table A.1 shows the summary of the core performance parameters. Using the given thermal efficiencies in Table A.1, the normalized mass flow rates per electricity generation for one year (i.e., t/GWe-yr) were calculated and provided in Table A.2. The equations to calculate the normalized mass flow data are introduced in the FCDP calculation note for the EG37 [A.1]. Similarly, the normalized mass flow rates per electricity generation for one year (i.e., t/GWe-year) were calculated by using an artificial uniform thermal efficiency of 100%, which is equivalent to the mass flow data per thermal power generation for one year (i.e., t/GWt-yr). The results are provided in Table A.3.

Table A.1 Summary of Core Performance and Mass Flow of Fuel Cycle Option

	Stage 1 PWR (UOX)	Stage 2 – SFR (TRU/U, Th)	Stage 3 PWR (U-233/U)
Reactor Power, MW-thermal	3000.0	3000.0	3400.0
Reactor Power, MW-electric	1000.0	1200.0	1100.0
Thermal efficiency, %	33.3	40.0	32.4
Reactor Capacity Factor	0.90	0.90	0.90
Enrichment of fresh driver fuel, %			
U-235/U	4.21	-	-
U-233/HM	-	-	3.4
TRU/HM	-	28.6	-
Number of batches	3	3 - 4	3
Average burnup, %	5.21	4.90	5.66
Specific power density, MW/t	33.7	31.2	37.3
Fuel residence time, year	4.5	4.5 – 6.0	4.5
Cycle length, year	1.5	1.5	1.5
Fuel inventory, t	89.05	96.23	91.14
HM charge per batch, t	29.68	30.00	30.38
HM discharge per batch, t	28.14	28.53	28.66
Average charge mass fraction, %			
Th	-	59.60	-
U-233+Pa-233	-	-	3.42
U (other than U-233+Pa-233)	100.00	28.80	96.58
TRU	0.00	11.60	-
Average discharge mass fraction, %			
Th	-	56.00	-
U-233+Pa-233	-	2.61	0.35
U (other than U-233+Pa-233)	93.48	26.22	92.71
TRU	1.31	10.26	1.29
FP	5.21	4.91	5.66

Table A.2 Normalized Mass Flow Rates per for Original Thermal Efficiency Values

	Stage 1 PWR (UOX)	Stage 2 SFR (TRU/U, Th)	Stage 3 PWR (U-233/U)
Thermal efficiency, %	33.3	40.0	32.5
Natural resource, t/GWe-yr			
Th	-	11.03	-
U	188.1	-	-
Charge mass, t/GWe-yr			
Th	-	11.03	-
U233+Pa233	-	-	0.70
U (other than U233+Pa233)	21.90	5.35	19.81
TRU	0.00	2.14	-
Discharge, t/GWe-yr			
Th	-	10.37	-
U233+Pa233	-	0.48	0.07
U (other than U233+Pa233)	20.47	4.86	19.02
TRU	0.29	1.90	0.26
FP	1.14	0.91	1.16
Mass balance, t/GWe-yr			
Th	-	-0.66	-
U233+Pa233	-	0.48	-0.63
U (other than U233+Pa233)	-1.43	-0.49	- 0.79
TRU	0.29	-0.24	0.26
FP	1.14	0.91	1.16

Table A.3 Normalized Mass Flow Rates for Thermal Power Generation

	Stage 1 PWR (UOX)	Stage 2 SFR (TRU/U, Th)	Stage 3 PWR (U-233/U)
Thermal efficiency, %	100.0	100.0	100.0
Natural resource, t/GWt-yr			
Th	-	4.41	-
U	62.71	-	-
Charge mass, t/GWt-yr			
Th	-	4.41	-
U233+Pa233	-	-	0.23
U (other than U233+Pa233)	7.30	2.14	6.41
TRU	0.00	0.86	-
Discharge, t/GWt-yr			
Th	-	4.15	-
U233+Pa233	-	0.19	0.02
U (other than U233+Pa233)	6.82	1.94	6.15
TRU	0.10	0.76	0.09
FP	0.38	0.36	0.38
Mass balance, t/GWt-yr			
Th	-	-0.26	-
U233+Pa233	-	0.19	-0.20
U (other than U233+Pa233)	-0.48	-0.20	- 0.26
TRU	0.10	-0.10	0.09
FP	0.38	0.36	0.38

The power-sharing fraction of each stage was calculated to ensure TRU and U-233 mass balances between the stages. At the equilibrium state, the recovered TRU mass from stages 1, 2, and 3 should be balanced to the required TRU mass in stage 2,

$$F_{EL}^{1} \times RM_{TRU}^{1} + F_{EL}^{3} \times RM_{TRU}^{3} = F_{EL}^{2} \times \Delta M_{TRU}^{2}, \tag{16}$$

where,

F_{EL}^{i} = power sharing fraction of stage *i*,

$\Delta M_{TRU}^{i} = FM_{TRU}^{i} - RM_{TRU}^{i}$ = external TRU feed for stage *i*,

$RM_{TRU}^{i} = (1 - L_{sep}^{i}) \times DM_{TRU}^{i}$ = recovered TRU mass from stage *i*,

$FM_{TRU}^{i} = \dfrac{CM_{TRU}^{i}}{(1 - L_{fuel}^{i})}$ = TRU mass for fuel fabrication of stage *i*,

DM_{TRU}^{i} = discharge TRU mass from stage *i*,

CM_{TRU}^{i} = charge TRU mass to stage *i*,

L_{sep}^{i} = separation loss rate of stage *i*,

L_{fuel}^{i} = fabrication loss rate of stage *i*.

Similarly, the recovered U-233 mass from stages 2 and 3 should balance the required U-233 mass in stage 3;

$$F_{EL}^{2} \times RM_{U3}^{2} = F_{EL}^{3} \times \Delta M_{U3}^{3}, \tag{17}$$

where,

$\Delta M_{U3}^{i} = FM_{U3}^{i} - RM_{U3}^{i}$ = external U-233 feed for stage *i*,

$RM_{U3}^{i} = (1 - L_{sep}^{i}) \times DM_{U3}^{i}$ = recovered U-233 mass from stage *i*,

$FM_{U3}^{i} = \dfrac{CM_{U3}^{i}}{(1 - L_{fuel}^{i})}$ = TRU mass for fuel fabrication of stage *i*,

DM_{U3}^{i} = discharge U-233 mass from stage *i*,

CM_{U3}^{i} = charge U-233 mass to stage *i*.

In addition, the total fraction of electricity generation of each stage should be unity,

$$F_{EL}^{1} + F_{EL}^{2} + F_{EL}^{3} = 1. \tag{18}$$

Using the three Eqs (16), (17) and (18), the power sharing fractions for the different thermal efficiencies were calculated and the results are provided in Table A.4 and A.5. The power sharing fractions calculated by the mass balance equations are exactly the same to the values calculated by using the Eq (14) (Compare power sharing fractions in Tables 1 and A.5).

Table A.4 Power Sharing Fractions with Original Thermal Efficiencies of 33.3, 40.0 and 32.4%

	Stage 1	Stage 2	Stage 3
Recovered (*RM*) mass, t/GWe-yr			
TRU	0.29	1.90	0.26
U-233	-	0.48	0.07
External feed (*ΔM*), t/GWe-yr			
TRU	-	0.27	-
U-233	-	-	0.64
Electricity sharing, %	11.9	50.1	38.0

Table A.5 Power Sharing Fractions with Thermal Power Generation

	Stage 1	Stage 2	Stage 3
Recovered (*RM*) mass, t/GWe-yr			
TRU	0.095	0.758	0.085
U-233	-	0.193	0.023
External feed (*ΔM*), t/GWe-yr			
TRU	-	0.107	-
U-233	-	-	0.206
Electricity sharing, %	12.8	45.0	45.2

At the equilibrium state, the normalized mass flow data of each stage was calculated such that the total electricity capacity of the whole nuclear fleet becomes 100 GWe-yr or 100 GWt-yr.

The mass flow data for stage 1 were obtained from

- Required natural uranium; $M^1_{NU} = \bar{M}^1_{NU} \times E^1$,
- Charge fuel (which is LEU); $M^1_{CF} = \bar{M}^1_{CF} \times E^1$,
- Loss in the fuel fabrication; $M^1_{loss,fuel} = M^1_{CF} \times L^1_{fuel}$,
- Depleted uranium (DU) ; $M^1_{DU} = M^1_{NU} - M^1_{CF} - M^1_{loss,fuel}$,
- Product from separation; $M^1_p = df^1_p \times \left(1 - L^1_{sep}\right) M^1_{DF}$,
- Loss in the fuel separation; $M^1_{loss,sep} = M^1_{DF} \times L^1_{sep}$,

where

$\bar{M}^1_{NU}$ = natural uranium mass to produce unit electricity in stage 1,

E^i = power generation from stage *i*, which is $100 \times F^i_{EL}$,

$\bar{M}^i_{CF}$ = normalized charge fuel mass of stage *i* (see Tables A.2 and A.3),

M^i_{DF} = discharge fuel mass from the stage *i* (which is identical to M^i_{CF}),

df^i_p = product-wise mass fraction of stage *i* discharge fuel (see Tables A.2 and A.3).

The mass flow data for stage 2 were obtained from

- Charge fuel; $M^2_{CF} = \bar{M}^2_{CF} \times E^2$,
- Loss in the fuel fabrication; $M^2_{loss,fuel} = L^2_{fuel} \times M^2_{CF}$,
- Product from separation; $M^2_p = df^2_p \times \left(1 - L^2_{sep}\right) M^2_{DF}$,
- Loss in the fuel separation; $M^2_{loss,sep} = M^2_{DF} \times L^2_{sep}$,
- Required natural thorium; $\Delta M^2_{Th} = \left(1 + L^2_{fuel}\right) M^2_{\text{Charge,Th}} - M^2_{\text{Recoverd,Th}}$,

where

$M^2_{\text{Charge,Th}}$ = thorium mass in charge fuel of stage 2 (= $cf^2_{Th} M^2_{CF}$),

$M^2_{\text{Recovered,Th}}$ = recovered thorium mass from stage 2 (= $df^2_{Th} M^2_{DF}$),

cf^2_{Th} = thorium mass fraction of stage 2 charge fuel (see Tables A.2 and A.3).

The mass flow data for stage 3 were obtained from

- Charge fuel; $M^3_{CF} = \bar{M}^3_{CF} \times E^3$,
- Loss in the fuel fabrication; $M^3_{loss,fuel} = L^3_{fuel} \times M^3_{CF}$,
- Product from separation; $M^3_p = df^3_p \times \left(1 - L^3_{sep}\right) M^3_{DF}$,
- Loss in the fuel separation; $M^3_{loss,sep} = M^3_{DF} \times L^3_{sep}$.

Using the equations, the normalized metal mass flow data of the primary heavy metal and fission products were calculated and the results are provided in Tables A.6 and A.7. In these tables, the signs (-) and (+) indicate the feed and production to or from each technology category, respectively.

For the given thermal efficiencies (i.e., 33.3%, 40.0% and 32.4% for the first, second, and third stages, respectively), the fuel cycle option requires natural uranium and thorium quantities of 2,238.0 and 40.4

tons, while the required natural uranium and thorium are 805.7 tons and 14.5 tons, respectively, for the thermal power generation case. Thus, the mass flow data for that case decrease uniformly to 36% of the original values, which is exactly the same as the value calculated by using the mass renormalization factor of Eq. (15), i.e., 0.36 (see Table 1 in the main body of the report). All other values in the Mass Flow Data table show the same renormalization value.

In this work, the renormalization formulas were validated only for the three-stage representative fuel cycle option for EG37. However, the formulas could be validated for other representative fuel cycle options by using the approach provided in this Appendix.

Reference

A.1 T. K. Kim, E. Hoffman, and T. A. Taiwo, "Completion of Fuel Cycle Data Package System Datasheets for 2013 Evaluation and Screening," Argonne National Laboratory, ANL-FCT-333, May 2013.

Table A.6 Normalized Mass Flow to Generate 100 GWe-yr with Original Thermal Efficiencies in FCDP System Datasheet (EG37)

Stage		1			2			3			Total
Technology		Fuel	NPPT	Sep/WF	Fuel	NPPT	Sep/WF	Fuel	NPPT	Sep/WF	
Electricity, GWe-yr		11.9			50.1			38.0			100.0
Feed or product of nuclear materials (metric ton) [a]											
Natural resource	U	-2,238									-2,238.0
	Th				-40.4						-40.0
Products from fuel or NPPT technology	DU	1,976.9						-37.5			1,939.4
	Th				552.4	-552.4					0.0
	U-233 [b]							26.7	-26.7		0.0
	U [c]	260.5	-260.5		267.7	-267.7		753.6	-753.6		0.0
	TRU				107.2	-107.2			0.0		0.0
	DF		260.5	-260.5		927.3	-927.3		780.3	-780.3	0.0
Products from Sep/WF technology	Th				-513.1		514.1				1.0
	U-233 [b]						24.0	-26.7		2.7	0.0
	U [c]			241.1	-268.2		240.7	-717.6		716.1	212.1
	TRU			3.4	-107.4		94.2			9.9	0.1
	FP			13.4			45.1			43.7	102.2
Sum (=Loss) [d]		0.52		2.61	1.86		9.27	1.56		7.80	23.6

a) Mass flow in metric ton was normalized to produce 100.0 GWe-year from whole nuclear fleet. The signs (-) and (+) indicate the feed and production to or from each technology category, respectively.
b) U-233 and Pa-233
c) Uranium other than U-233 and Pa-233
d) Summation of each column indicates the loss from each technology per each stage.

Table A.7 Normalized Mass Flow to Generate 100 GWt-yr (EG37)

Stage		1			2			3			Total
Technology		Fuel	NPPT	Sep/WF	Fuel	NPPT	Sep/WF	Fuel	NPPT	Sep/WF	
Electricity, GWt-yr		12.8			45.0			42.2			100.0
Feed or product of nuclear materials (metric ton) [a]											
Natural resource	U	-805.7									-805.7
	Th				-14.5						-14.5
Products from fuel or NPPT technology	DU	711.7						-13.5			698.2
	Th				198.3	-198.3					0.0
	U-233 [b]							9.6	-9.6		0.0
	U [c]	93.8	-93.8		96.1	-96.1		270.6	-270.6		0.0
	TRU				38.5	-38.52			0.0		0.0
	DF		93.8	-93.8		333.0	-333.0		280.1	-280.1	0.0
Products from Sep/WF technology	Th				-184.2		184.6				0.4
	U-233 [b]						8.6	-9.6		1.0	0.0
	U [c]			86.8	-96.3		86.4	-257.6		257.1	76.4
	TRU			1.2	-38.6		33.8			3.6	0.0
	FP			4.8			16.2			15.7	36.7
Sum (=Loss) [d]		0.19		0.94	0.67		3.33	0.56		2.80	8.5

a) Mass flow in metric ton was normalized to produce 100.0 GWt-year from whole nuclear fleet. The signs (-) and (+) indicate the feed and production to or from each technology category, respectively.
b) U-233 and Pa-233
c) Uranium other than U-233 and Pa-233
d) Summation of each column indicates the loss from each technology per each stage.