ANL-HEP-TR-09-112

Argonne NATIONAL LABORATORY

DC 12 m Telescope
Preliminary Calculations
Investigation of Elevation Axis Position

High Energy Physics Division

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Investigation of Elevation Axis Position

by
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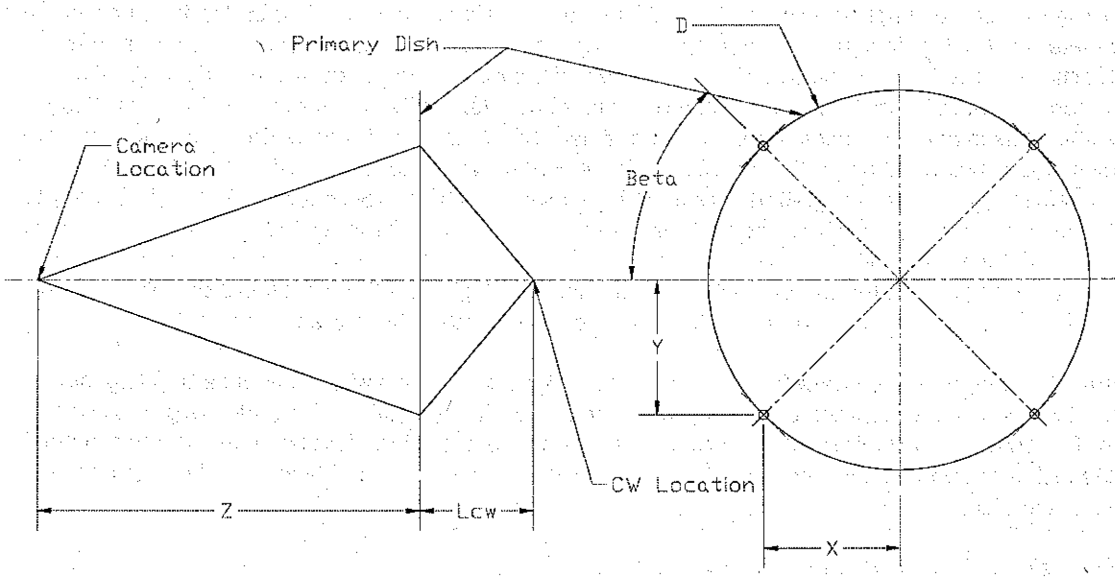
November 2009

This paper examines some simple calculations of a 2D model of a telescope in order to understand how different design parameters affect the design. For the design of a telescope it is assumed that we need a design that mimizes deflections of the dish and also minimizes the size of the motors and torques needed to rotate in elevation. A common belief is that a lighter dish and minimum counterweight is desireable. However, these calculations show this is not necessarily true. The torque needed for rotation depends on the moment of inertia and if the telescope is balanced about the elevation axis. A light dish with no CW requires that the elevation axis be several meters in front of the dish (8-9m) in order to be balanced. This is not practical from a structural point of view. If the elevation axis is only 2m in front of the dish and there is no counterweight then the telescope will be unbalanced and the torques required will be very high -- much higher than the torques needed only to overcome inertia. A heavy dish though can act as its own counterweight and the elevation axis only has to be 2-3m in front of the dish in order to achieve a balanced telescope. (see sections 5 and 6 below)

Also, the struts that support the camera from the dish place a load on the dish which will put a bending moment on the dish. This bending moment will deform the dish and require it to be stiffer. A counterweight structure performs two functions. First, it allows the telescope to be balanced about the elevation axis. Second, it applies a force on the dish that opposes the forces from the camera struts, thereby reducing the bending moment and deformations of the dish.

The main conclusions of this study are:
-- with the elevation axis in front of the dish a heavier dish reduces the need for a counterweight
--The moment of inertia is reduced if a lighter dish is used but the reduction is not as great as the reduction in weight of the dish. For example, a 75% reduction in dish weight results in only a 15% reduction in the inertia. This is due the higher counterweight that is needed with a lighter dish. (see section 4.0 below)
--The use of a counterweight allows the telescope to be balanced about the elevation axis so that the torque on the elevation motor is only due to inertia.
--The use of a counterweight counterbalances the forces placed on the dish from the struts that support the camera, thereby reducing the bending moment on the dish and dish deformations
--If a counterbalance is not used then the telescope will not be balanced about the elevation axis because the balance poit is so far in front of the dish to be practical. If the telescope is not balanced then the torques needed for rotatino will be very high, much higher than if the telescope was balanced.

The calculations shown below and the values used are simply meant to be representative and not exact values that would be used in a telescope. The purpose of these calculations and the values used for variables is to undertand the interactions between the many variables that influence the design.



1.0 Define Inputs

1.1 Geometry

ksi := 1000psi   kips := 1000lbf   ton := 2000lbf

$\beta 1 := 60\text{deg}$ Angular position of camera supports on Main Ring

E := 30000ksi

$\text{Idish} := 9.501 \cdot 10^{4} \cdot \text{kg} \cdot \text{m}^{2}$ Moment of Inertia of dish about the elevation axis

FD := 1.5   F/D ratio

D := 12m   Diameter of Dish

$z := D \cdot FD$ Distance from dish to camera

$z = 18.00 \cdot m$

Wc := 2.5ton   Weight of Camera

Wdish := 20ton   Weight of Dish

Lcw := 15ft   Distance from Dish to counterweights

$\rho s := .283 \frac{\text{lbf}}{\text{in}^{3}}$ Weight Density of Steel

$$\frac{\rho s}{g} = 7833.41 \cdot \frac{kg}{m^3}$$

1.2 Calculate positions of Camera Supports and Length of Camera Supports

$x(\beta) := \frac{D}{2} \cdot \cos(\beta)$ $\quad x(\beta 1) = 3.00\,m$ $\quad$ X position of support on Main ring

$x(\beta 1) = 3.00 \cdot m$

$y(\beta) := \frac{D}{2} \cdot \sin(\beta)$ $\quad y(\beta 1) = 5.20\,m$ $\quad$ Y position of support on Main ring

$y(\beta 1) = 5.20 \cdot m$

$L(\beta) := \sqrt{x(\beta)^2 + y(\beta)^2 + z^2}$ $\quad L(\beta 1) = 18.97\,m$ $\quad$ Length of Camera supports

$L(\beta 1) = 18.97 \cdot m$

$$L2(\beta, Lcw) := \begin{cases} (0m) & \text{if } Lcw = 0m \\ \sqrt{x(\beta)^2 + y(\beta)^2 + Lcw^2} & \text{otherwise} \end{cases}$$

$L2(\beta 1, Lcw) = 24.75 \cdot ft$ $\quad$ Length of counter weight support

$L2(\beta 1, Lcw) = 7.54 \cdot m$

1.3 Calculate the Total Weight Supported at Camera including camera weight and support weights.

$A := 6.59 in^2$ $\quad$ Initial guess at cross sectional area of supports

$W(\beta) := Wc + 2 \cdot L(\beta) \cdot \rho s \cdot A$ $\quad W(\beta 1) = 7786.25 \cdot lbf$ $\quad$ Total weight supported at Camera

$W(\beta 1) = 34634.95 \cdot N$

1.4 Calculate the angle from supports to X, Y, Z

$\alpha x(\beta) := acos\left(\frac{x(\beta)}{L(\beta)}\right)$ $\quad \alpha x(\beta 1) = 80.90 \cdot deg$

$\alpha y(\beta) := acos\left(\frac{y(\beta)}{L(\beta)}\right)$ $\quad \alpha y(\beta 1) = 74.11 \cdot deg$

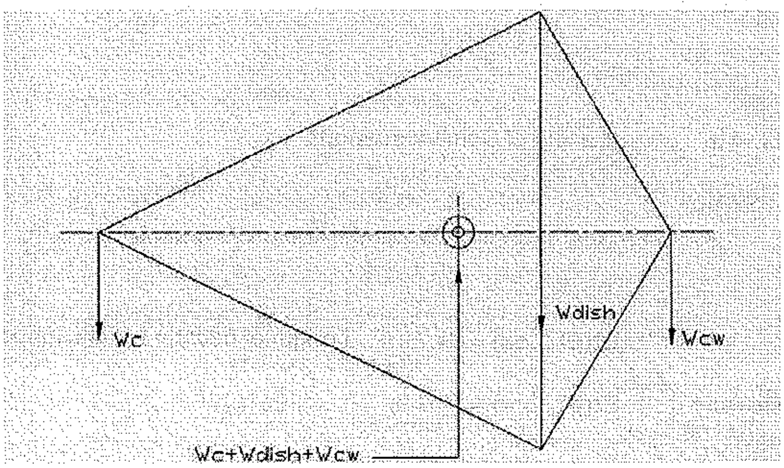
$\alpha z(\beta) := acos\left(\frac{z}{L(\beta)}\right)$ $\quad \alpha z(\beta 1) = 18.43 \cdot deg$

$\alpha cw(\beta, Lcw) := acos\left(\dfrac{Lcw}{L2(\beta, Lcw)}\right)$

$\alpha cw(\beta 1, Lcw) = 52.69 \cdot deg$

2.0 Calculate the Required Counter Weight to have Zero moment about the positioner rotation point

Calculate the required counterweight by summing the moments about the elevation axis. For simplicity assume that the weight of the beams is acting halfway along Lc and Lcw.



$qc := \rho s \cdot A$ $\qquad qc = 22.38 \cdot \dfrac{lbf}{ft}$ $\qquad qc = 326.61 \cdot \dfrac{N}{m}$ Weight per length of camera struts

$L1 := L(\beta 1)$ $\qquad L1 = 18.97 \cdot m$

$qcw := 58.03 \dfrac{lbf}{ft}$ $\qquad qcw = 846.88 \cdot \dfrac{N}{m}$ Weight per length of CW struts

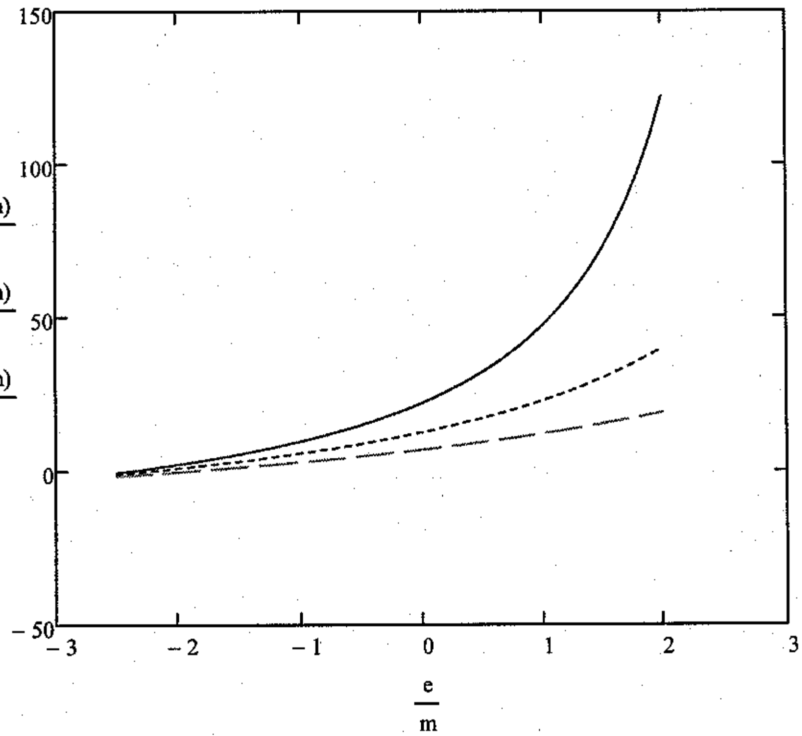
$$Wcw(e, Lcw) := \frac{Wc \cdot (z + e) + 4 \cdot qc \cdot L1 \cdot \left(\dfrac{z}{2} + e\right) + Wdish \cdot e - 4 \cdot qcw \cdot L2(\beta 1, Lcw) \cdot \left(\dfrac{Lcw}{2} - e\right)}{Lcw - e}$$ Counterweight as a function of e and Lcw

$Wcw(-1.5m, 5m) = 3.12 \cdot ton$ Required counterweigt for e=-1.5m

$Lcw := 0m$ $\qquad L2(\beta 1, Lcw) = 0.00\,m$

$Wcw(-3.57m, Lcw) = -5.66 \cdot ton$

$e := -2.5m, -2.4m .. 2.m$



$Wcw(2m, 5m) = 39.72 \cdot ton$

$Wcw(1m, 5m) = 22.73 \cdot ton$

$Wcw(0m, 5m) = 12.53 \cdot ton$

$Wcw(-1.5m, 5m) = 3.12 \cdot ton$

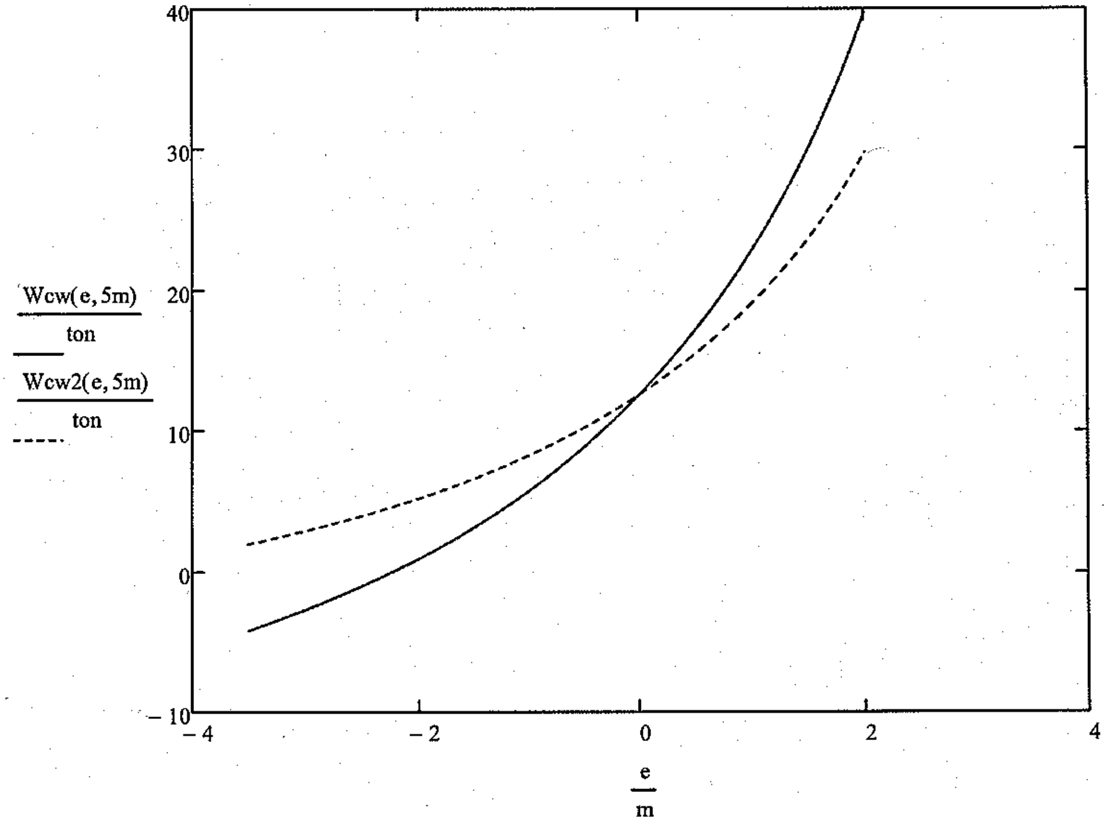
$Wcw(-2.5m, 5m) = -1.07 \cdot to$

The graph above shows that as the elevation axis moves from the back of the dish to the front of the dish the required counterweight declines.

What happens when the dish weight is reduced by 75%????

$$Wcw2(e, Lcw) := \frac{Wc \cdot (z + e) + 4 \cdot qc \cdot L1 \cdot \left(\frac{z}{2} + e\right) + 0.25 \cdot Wdish \cdot e - 4 \cdot qcw \cdot L2(\beta 1, Lcw) \cdot \left(\frac{Lcw}{2} - e\right)}{Lcw - e}$$

$e := -3.5m, -3.4m .. 2m$



If the elevation axis is in front of the dish then a heavy dish reduces the counterweight that is needed.

3.0 Calculate the forces in Truss Members

3.1 Forces in the arms supporting the camera

$\mathrm{Lcw} := 5\mathrm{m}$

$e := 0\mathrm{m}$

Calculate tension force in top members supporting the camera when telescope is horizontal

$$\mathrm{F1z}(\beta) := \frac{W(\beta)\cdot z + 4\cdot \mathrm{qc}\cdot \mathrm{L1}\cdot\left(\frac{z}{2}\right)}{4\cdot y(\beta)}$$

$\mathrm{F1z}(\beta 1) = 9.16\cdot\mathrm{kips}$

$$F1(\beta) := \frac{F1z(\beta)}{\cos(\alpha z(\beta))}$$

$F1(\beta1) = 9.65 \cdot kips$ Axial force in top members

Calculate the compression force in the bottom member supporting the camera

$F2z(\beta) := F1z(\beta)$ $F2z(\beta1) = 9.16 \cdot kips$

$$F2(\beta) := \frac{F2z(\beta)}{\cos(\alpha z(\beta))}$$

$F2(\beta1) = 9.65 \cdot kips$ Axial compression force in bottom member

3.2 Forces in the arms supporting the CW

Calculate tension force in top members supporting the counter weight when telescope is horizontal

$Lcw := 5m$

$e := 0m$

$$F3z(\beta, e, Lcw) := \frac{Wcw(e, 5m) \cdot Lcw + 4 \cdot qcw \cdot L2(\beta1, Lcw) \cdot \left(\frac{Lcw}{2}\right)}{4 \cdot y(\beta)}$$

$F3z(\beta1, e, Lcw) = 6.74 \cdot kips$

$$F3(\beta, e, Lcw) := \frac{F3z(\beta, e, Lcw)}{\cos(\alpha cw(\beta, Lcw))}$$

$F3(\beta1, e, Lcw) = 10.53 \cdot kips$ Axial force in top members

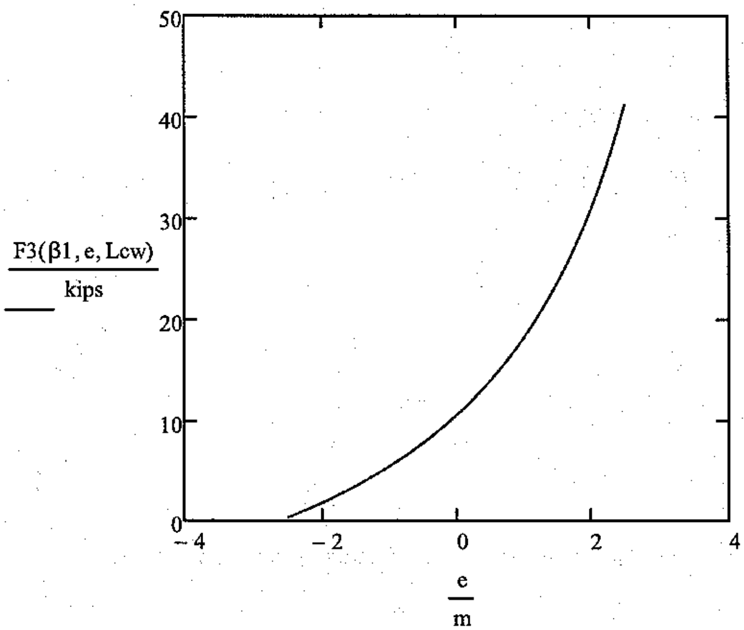
Calculate the compression force in the bottom member supporting the camera

$F4z(\beta, e, Lcw) := F3z(\beta, e, Lcw)$ $F4z(\beta1, e, Lcw) = 6.74 \cdot kips$

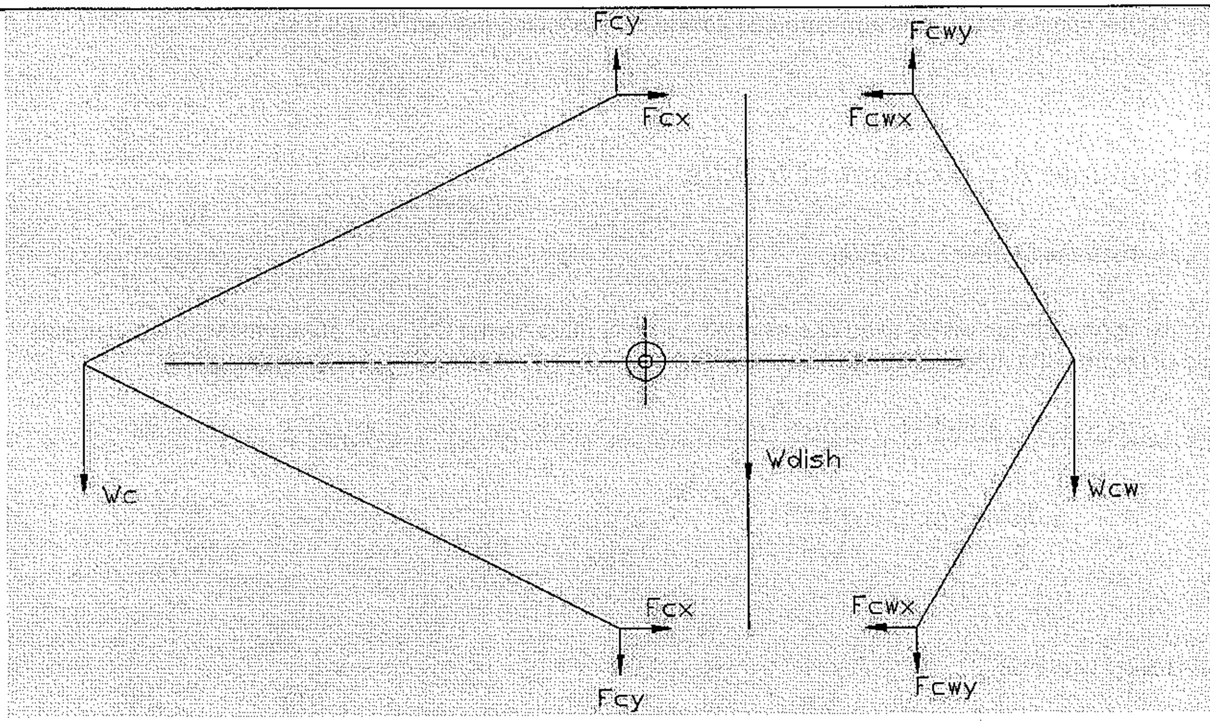
$$F4(\beta, e, Lcw) := \frac{F4z(\beta, e, Lcw)}{\cos(\alpha cw(\beta, Lcw))}$$

$F4(\beta1, e, Lcw) = 10.53 \cdot kips$ Axial compression force in bottom member

$e := -2.5m, -2.4m .. 2.5m$



3.3 Check Equilibrium at the points where the camera and counterweight truss members connect



$e := 0m$

$Lcw := 5m$

$Fcx := F1(\beta 1)\cdot \cos(\alpha z(\beta 1)) = 4.58\cdot ton$

$Fcwx := F3(\beta 1, e, Lcw)\cdot \cos(\alpha cw(\beta 1, Lcw)) = 3.37\cdot ton$

So for e=0 the horizontal forces from the truss members are equal. Therefore, the truss members do no create any bending moment on the dish.

What happens when e is not equal to zero?

$e := -2m$

$Lcw := 5m$

$Fcx := F1(\beta 1)\cdot \cos(\alpha z(\beta 1)) = 4.58\cdot ton$

$Fcwx := F3(\beta 1, e, Lcw)\cdot \cos(\alpha cw(\beta 1, Lcw)) = 0.57\cdot ton$

$Fcx - Fcwx = 4.01\cdot ton$

When e is not zero then an imbalance occurs. The resultant horizontal force applied to the dish by the support struts for the camera and CW is non zero and acts to put a bending moment on the dish.

4.0 Evaluate the Moment of Inertia

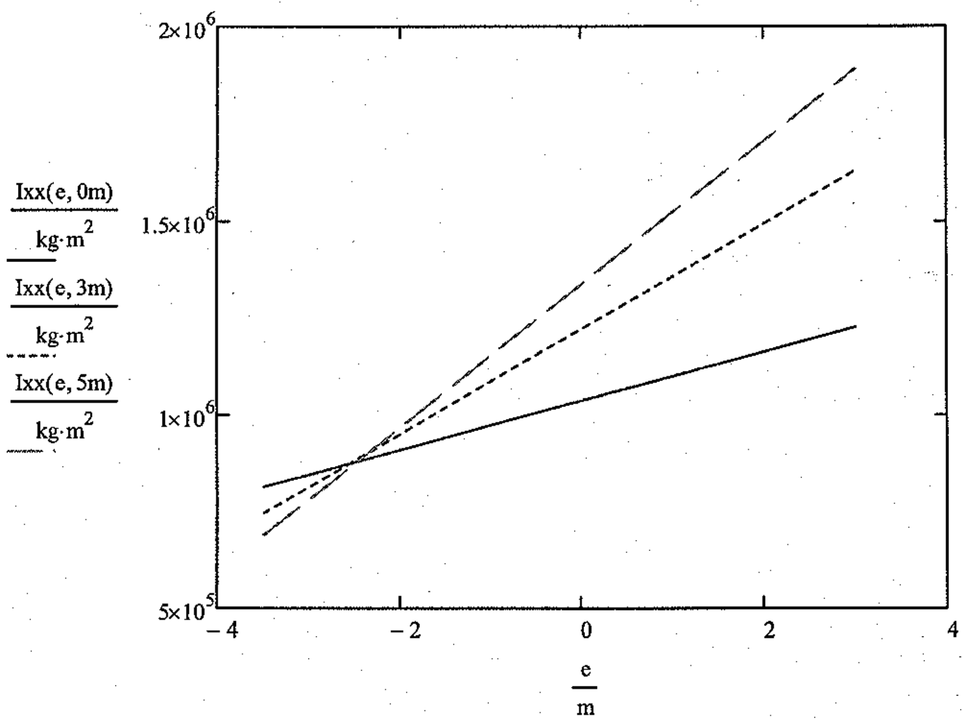
4.1 Calculate the Moment of Inertia

A simple approximation will be used of a assuming a point mass for the CW and camera masses and using a value of Ixx from the model of the dish.

$$Ixx(e,Lcw) := Idish + \frac{Wdish}{g}\cdot e^2 + \frac{Wcw(e,Lcw)}{g}\cdot(Lcw-e)^2 + \frac{Wc}{g}\cdot(z+e)^2 + 4\frac{qc}{g}\cdot L1\cdot\left(\frac{z}{2}+e\right)^2 \ldots$$
$$+4\cdot\frac{qcw}{g}\cdot L2(\beta 1,Lcw)\cdot\left(\frac{Lcw}{2}-e\right)^2$$

$e := -3.5m, -3.4m .. 3m$

$Ixx(2m,5m) = 1.71\times 10^6\cdot kg\cdot m^2$



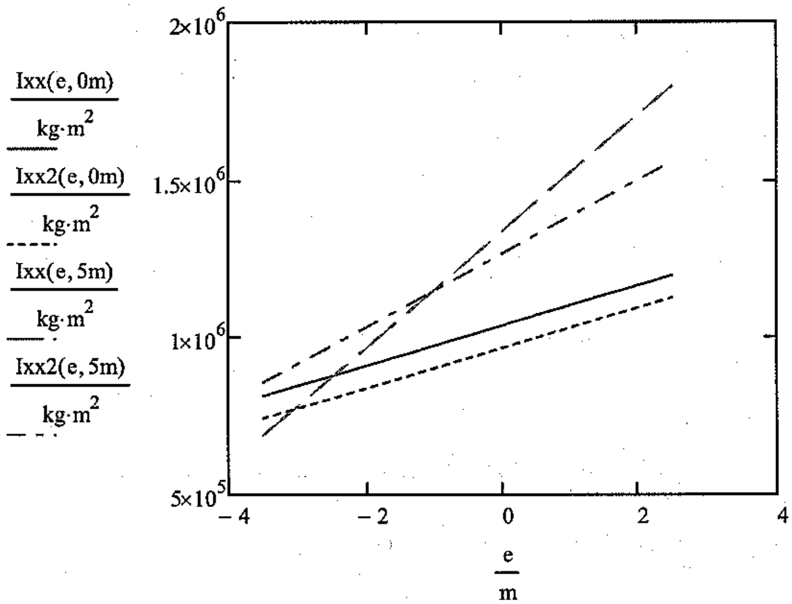
4.2 How do Wcw and Ixx change if the weight of the dish is reduced by 75%

$\mathrm{Wdish} = 20.00 \cdot \mathrm{ton}$

$$\mathrm{Wcw2}(e, \mathrm{Lcw}) := \frac{\mathrm{Wc}\cdot(z+e) + 4\cdot \mathrm{qc}\cdot \mathrm{L1}\cdot\left(\frac{z}{2}+e\right) + 0.25\cdot \mathrm{Wdish}\cdot e - 4\cdot \mathrm{qcw}\cdot \mathrm{L2}(\beta 1, \mathrm{Lcw})\cdot\left(\frac{\mathrm{Lcw}}{2} - e\right)}{\mathrm{Lcw} - e}$$

$$\mathrm{Ixx2}(e, \mathrm{Lcw}) := .25\cdot \mathrm{Idish} + \frac{.25\cdot \mathrm{Wdish}}{g}\cdot e^2 + \frac{\mathrm{Wcw2}(e,\mathrm{Lcw})}{g}\cdot(\mathrm{Lcw}-e)^2 + \frac{\mathrm{Wc}}{g}\cdot(z+e)^2 \ldots$$
$$+\left[4\frac{\mathrm{qc}}{g}\cdot \mathrm{L1}\cdot\left(\frac{z}{2}+e\right)^2 + 4\cdot\frac{\mathrm{qcw}}{g}\cdot \mathrm{L2}(\beta 1, \mathrm{Lcw})\cdot\left(\frac{\mathrm{Lcw}}{2}-e\right)^2\right]$$

$e := -3.5\mathrm{m}, -3.4\mathrm{m} .. 2.5\mathrm{m}$



$\mathrm{Ixx}(-2\mathrm{m}, 0\mathrm{m}) = 9.07\times 10^5\cdot \mathrm{kg}\cdot \mathrm{m}^2$

$\mathrm{Ixx2}(-2\mathrm{m}, 0\mathrm{m}) = 8.36\times 10^5\cdot \mathrm{kg}\cdot \mathrm{m}^2$

$\mathrm{Ixx}(-2\mathrm{m}, 5\mathrm{m}) = 9.66\times 10^5\cdot \mathrm{kg}\cdot \mathrm{m}^2$

$\mathrm{Ixx2}(-2\mathrm{m}, 5\mathrm{m}) = 1.03\times 10^6\cdot \mathrm{kg}\cdot \mathrm{m}^2$

$\frac{\mathrm{Ixx}(-2\mathrm{m}, 0\mathrm{m})}{\mathrm{Ixx}(-2\mathrm{m}, 5\mathrm{m})} = 0.94$

$\frac{\mathrm{Ixx2}(-2\mathrm{m}, 0\mathrm{m})}{\mathrm{Ixx}(-2\mathrm{m}, 0\mathrm{m})} = 0.92$

$\frac{\mathrm{Ixx2}(-2\mathrm{m}, 5\mathrm{m})}{\mathrm{Ixx}(-2\mathrm{m}, 5\mathrm{m})} = 1.07$

$\text{Ixx}(1\text{m}, 0\text{m}) = 1.10 \times 10^{6} \cdot \text{kg} \cdot \text{m}^{2}$

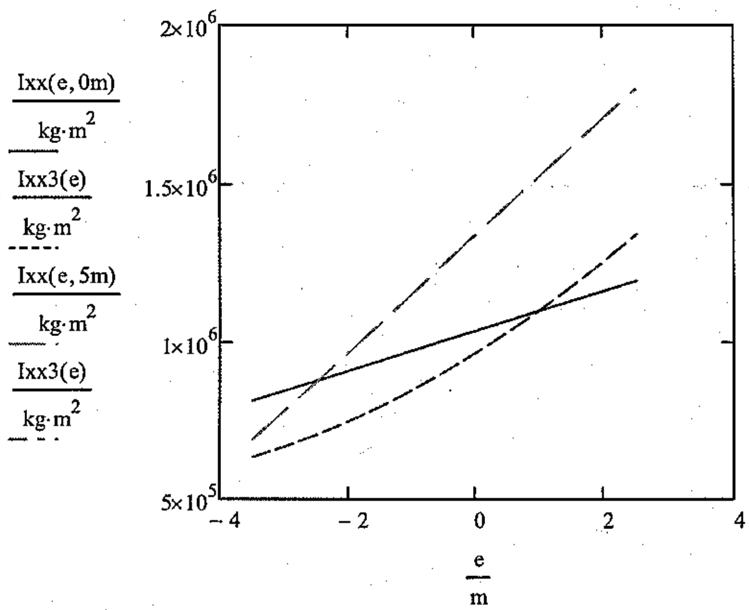
$\text{Ixx2}(1\text{m}, 0\text{m}) = 1.03 \times 10^{6} \cdot \text{kg} \cdot \text{m}^{2}$

4.3 What is the affect of in the Inertia, Ixx, if there is no counterweight and a lighter dish is used??

$\text{Wdish} := 20\text{ton}$

$$\text{Ixx3}(e) := .25 \cdot \text{Idish} + \frac{0.25 \cdot \text{Wdish}}{g} \cdot e^{2} + \frac{\text{Wc}}{g} \cdot (z + e)^{2} + \left[4 \frac{\text{qc}}{g} \cdot \text{L1} \cdot \left(\frac{z}{2} + e \right)^{2} \right]$$

$e := -3.5\text{m}, -3.4\text{m} .. 2.5\text{m}$



$\text{Ixx}(-2\text{m}, 0\text{m}) = 9.07 \times 10^{5} \cdot \text{kg} \cdot \text{m}^{2}$

$\text{Ixx3}(-2\text{m}) = 7.46 \times 10^{5} \cdot \text{kg} \cdot \text{m}^{2}$

$\text{Ixx}(-2\text{m}, 5\text{m}) = 9.66 \times 10^{5} \cdot \text{kg} \cdot \text{m}^{2}$

$\text{Ixx3}(-2\text{m}) = 7.46 \times 10^{5} \cdot \text{kg} \cdot \text{m}^{2}$

$\frac{\text{Ixx}(-2\text{m}, 0\text{m})}{\text{Ixx}(-2\text{m}, 5\text{m})} = 0.94$

$\frac{\text{Ixx3}(-2\text{m})}{\text{Ixx}(-2\text{m}, 0\text{m})} = 0.82$

$\frac{\text{Ixx3}(-2\text{m})}{\text{Ixx}(-2\text{m}, 5\text{m})} = 0.77$

5.0 Calculate Torques on Elevation Motor

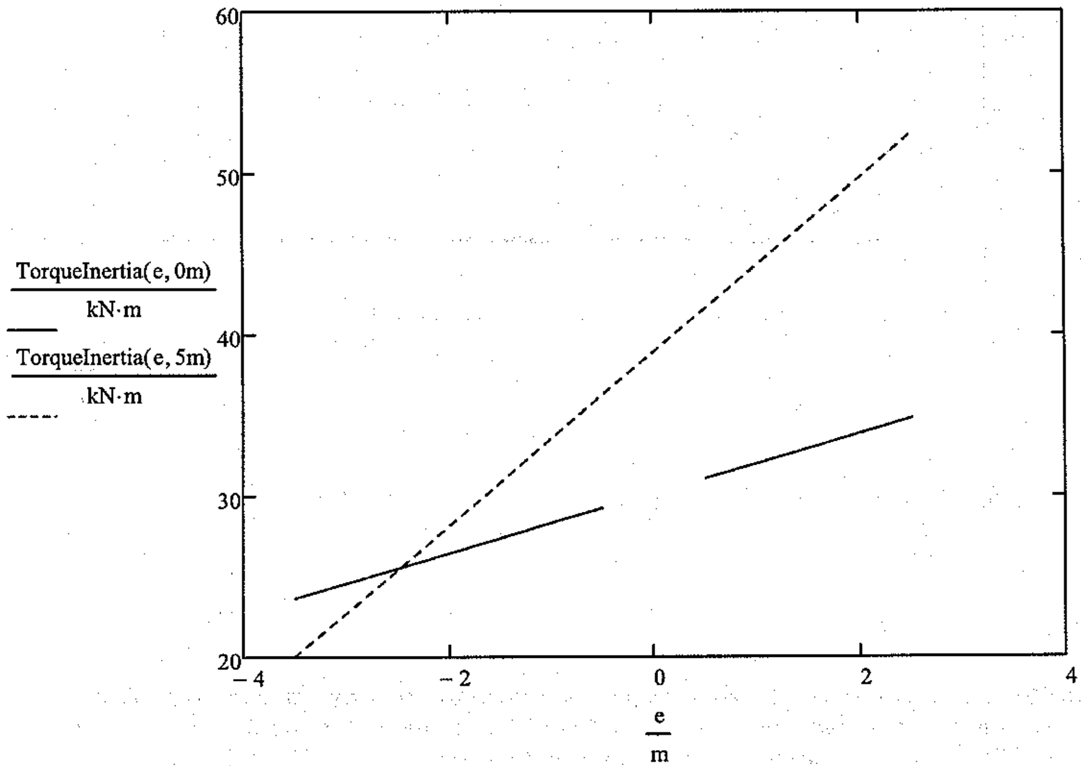
When a counterbalance is used the torque needed for rotation is simply due to inertia.

$\alpha := 1.67 \frac{deg}{s^2}$ Angular acceleration

$Wdish = 20.00 \cdot ton$

$TorqueInertia(e, Lcw) := Ixx(e, Lcw) \cdot \alpha$

$e := -3.5m, -3.m .. 2.5m$



$TorqueInertia(1m, 0m) = 32.01 \cdot kN \cdot m$ $TorqueInertia(1m, 0m) = 32.007 \cdot kN \cdot m$

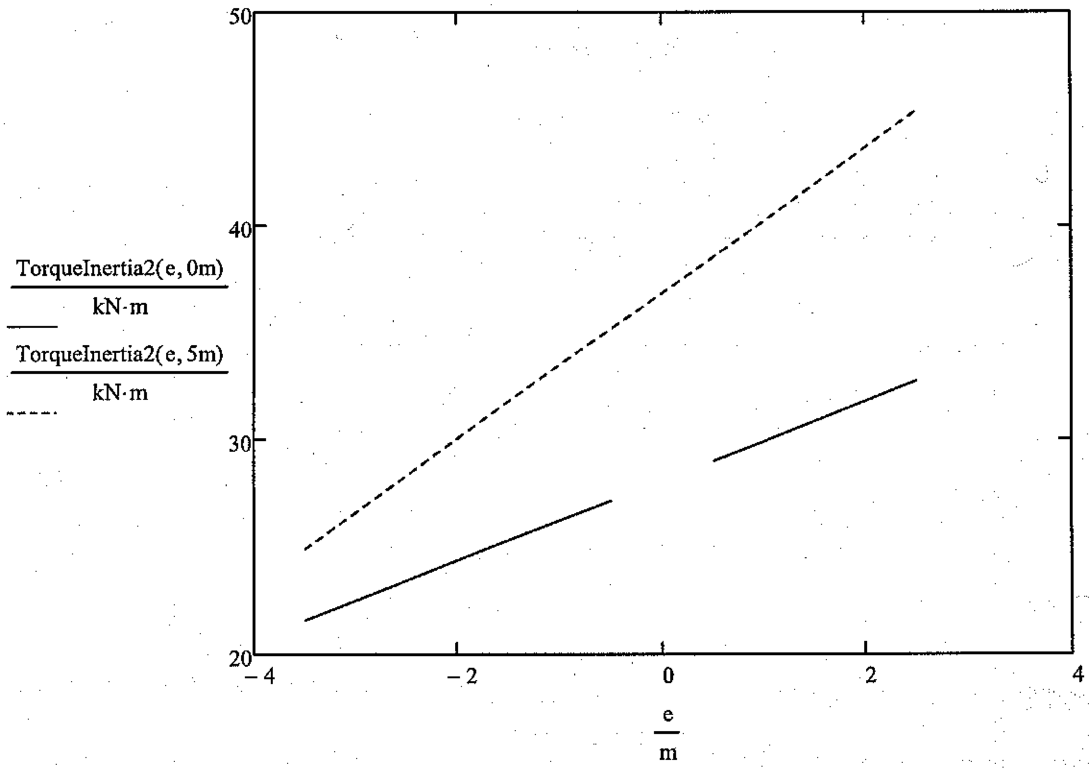
$TorqueInertia(-2m, 5m) = 28.143 \cdot kN \cdot m$ $TorqueInertia(0m, 5m) = 38.928 \cdot kN \cdot m$

5.2 How does the torque due to Inertia only change with a lighter dish?

$Wdish := 4ton$

$TorqueInertia2(e, Lcw) := Ixx2(e, Lcw) \cdot \alpha$

$e := -3.5m, -3.m .. 2.5m$



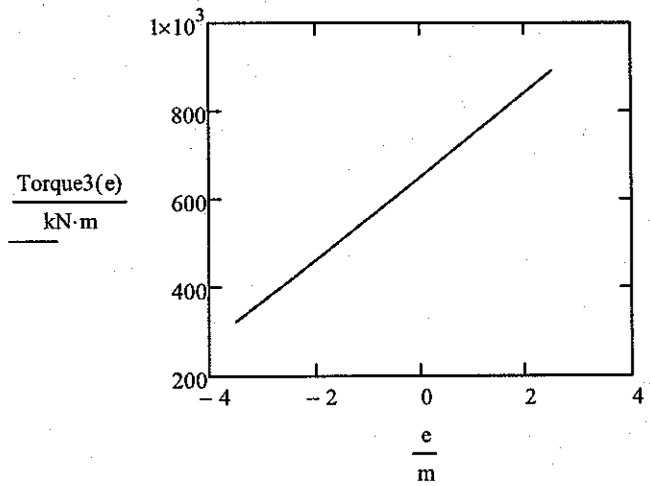
$\text{TorqueInertia2}(-2.5\text{m}, 0\text{m}) = 23.445\cdot\text{kN}\cdot\text{m}$ $\qquad$ $\text{TorqueInertia2}(1\text{m}, 0\text{m}) = 29.931\cdot\text{kN}\cdot\text{m}$

5.3 By definition the torque about the elevation axis is zero when a counterbalance is used. What is the torque when there is no counterbalance used and the dish is lighter? With no CW the elevation motors must resist the unbalances load since rotation is not about the center of gravity.

$\text{Wdish} := 20\text{ton}$

$$\text{Torque3}(e) := \text{Wc}\cdot(z + e) + 0.25\cdot\text{Wdish}\cdot e + 4\text{qc}\cdot\text{L1}\cdot\left(\frac{z}{2} + e\right) + \text{Ixx3}(e)\cdot\alpha$$

$e := -3.5\text{m}, -3.\text{m} .. 2.5\text{m}$



$\text{Torque3}(-3\text{m}) = 368.30\cdot\text{kN}\cdot\text{m}$ $\text{Torque3}(1\text{m}) = 747.00\cdot\text{kN}\cdot\text{m}$

6.0 Calculate the center of balance if there is no CW

6.1 Center of balance with a light 4 ton dish and no CW

$\text{Wdish} := 4\text{ton}$

$e := 8\text{m}$

Given

$$\text{Wc}\cdot(z + e) + 4\text{qc}\cdot\text{L1}\cdot\left(\frac{z}{2} + e\right) = -\text{Wdish}\cdot e$$

$e := \text{Find}(e)$

$e = -7.55\,\text{m}$

$$\text{Wc}\cdot(z + e) + 4\cdot\text{qc}\cdot\text{L1}\cdot\left(\frac{z}{2} + e\right) + \text{Wdish}\cdot e = -0.00\cdot\text{N}\cdot\text{m}$$

6.2 Center of balance with a 10 ton dish and no CW

$\text{Wdish} := 10\text{ton}$

$e := 8\text{m}$

Given

$$Wc\cdot(z+e)+4qc\cdot L1\cdot\left(\frac{z}{2}+e\right)=-Wdish\cdot e$$

$e := Find(e)$

$e = -4.58\,m$

$$Wc\cdot(z+e)+4\cdot qc\cdot L1\cdot\left(\frac{z}{2}+e\right)+Wdish\cdot e=0.00\cdot N\cdot m$$

6.3 Center of balance wtih a 20 ton dish and no CW

$Wdish := 20ton$

$e := 8m$

Given

$$Wc\cdot(z+e)+4qc\cdot L1\cdot\left(\frac{z}{2}+e\right)=-Wdish\cdot e$$

$e := Find(e)$

$e = -2.77\,m$

$$Wc\cdot(z+e)+4\cdot qc\cdot L1\cdot\left(\frac{z}{2}+e\right)+Wdish\cdot e=-0.00\cdot N\cdot m$$

7.0 Calculate Minimum values of Wcw and Ixx

$e := 0m$ $\qquad$ $Lcw := 1m$

$Wdish := 20ton$

Find the values of e and Lcw that minimize the counterweight

Given

$-3m < e < 2m$ $\qquad$ Possible range of e

$3m < Lcw < 10m$ $\qquad$ Possible range of Lcw

$$\begin{pmatrix} e \\ Lcw \end{pmatrix} := \text{Minimize}(Wcw, e, Lcw)$$

$emin := e$ $\quad emin = -3.00\,m$ Values of e and Lcw that Minimize Ixx

$Lcwmin := Lcw$ $\quad Lcwmin = 10.00\,m$

$Wcw(emin, Lcwmin) = -3.18 \cdot ton$ Minimum value of Wcw

$Ixx(emin, Lcwmin) = 6.303 \times 10^5 \cdot kg \cdot m^2$ Value of Ixx when Wcw is minimized

Find the values of e and Lcw that minimize the moment of inertia

$e := 1m$ $\quad Lcw := 2m$

Given

$-3m < e < 2m$ Possible range of e

$3m < Lcw < 10m$ Possible range of Lcw

$$\begin{pmatrix} e \\ Lcw \end{pmatrix} := \text{Minimize}(Ixx, e, Lcw)$$

$emin := e$ $\quad emin = -3.00\,m$ Values of e and Lcw that Minimize Ixx

$Lcwmin := Lcw$ $\quad Lcwmin = 10.00\,m$

$Wcw(emin, Lcwmin) = -3.18 \cdot ton$ Minimum value of Wcw

$Ixx(emin, Lcwmin) = 6.303 \times 10^5 \cdot kg \cdot m^2$ Value of Ixx when Wcw is minimized

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