An Empirical Correlation for E(J,T) of a Melt-Cast-Processed BSCCO-2212 Superconductor under Self Field

Y. S. Cha

Abstract—**An empirical correlation is developed for the electrical field strength E(J,T) of a melt-cast processed BSCCO-2212 superconductor. The empirical correlation is based, in part, on the theory of magnetic relaxation and on experimental data at 77 and 87 K. It is developed for temperatures in the range between 77 and 92 K, which is the range of interest for practical devices such as the superconducting fault current limiters. The general form of the correlation may be applicable to other high-T_c superconductors.**

Index Terms—**Fault current limiters, flux-creep resistivity, magnetic diffusion, magnetic relaxation**

I. INTRODUCTION

A basic property of superconductors is the electric field strength E, which is a function of the current density J, the temperature T, and the magnetic flux density B. The magnetic flux density B is the sum of the applied field and the self field due to the current density J. If there is no applied field, the electric field strength can be considered a function of the current density and temperature, E = E(J,T). In practical applications of high-T_c superconducting devices, information on E(J,T) is often needed over a range of current density and temperature. For example, in a resistive fault current limiter, the superconductor heats up substantially during a fault. Even for the so-called superconductor shielded-core reactor (SSCR), which is often known as the inductive fault current limiter, the superconductor tube heats up considerably during a fault. To accurately determine the voltage drop across the fault current limiter during a fault, a complete map of E as a function of J and T must be known. Recently, Cha [1-3] reported that thermal and magnetic diffusion is the mechanism for field penetration of a superconductor tube when it is subjected to a pulsed magnetic field. Similarly, thermal and magnetic diffusion is important for the SSCR because it is based on the shielding capability of a superconductor tube. Furthermore, thermal and magnetic diffusion is also important for the trapping of a magnetic field in a superconductor pellet that is using a pulsed current supply [4-6]. As pointed out by Cha [1], to model the coupled thermal and magnetic diffusion and understand how the magnetic field and temperature of the superconductor evolve during a transient, complete information on E(J,T) must be known, presumably from experimental measurement. However, such information is difficult to obtain because most researchers only measure the E/J characteristics at one or two temperatures (mostly at 77 K).

In this paper, we employ the data on E/J characteristics of a melt-cast processed BSCCO-2212 superconductor at two temperatures (77 and 87 K). Our main objective is to develop an empirical correlation for E(J,T), based on the experimental data at 77 and 87 K, which can then be used to calculate E from 77 K to the critical temperature of 90-92 K. The correlation can then be used to calculate the electric field strength E for a range of temperatures and current densities of interest for practical devices made of melt-cast processed BSCCO-2212 superconductors. The correlation we developed is based, in part, on the theory of magnetic relaxation [7], and its general form may be applicable to other high-T_c superconductors.

II. EXPERIMENTAL DATA

The experimental data were reported previously [8]. The superconductor is a melt-processed BSCCO rod made by Hoechst (now called Nexans). The diameter of the rod is 7.85 mm. The standard four-points measurement technique was employed for all of the tests. The distance between the voltage taps was 87 mm. Braided current leads were soldered to the ends of the sample and connected to a pulsed power supply. Pulsed current with a duration of 400 ms (square wave) was used in the experiment. Tests at 77 K were conducted in an open dewar containing liquid nitrogen; tests at 87 K were conducted in the same open dewar containing liquid argon.

Figure 1 shows the V/I characteristics of the melt-cast processed BSCCO-2212 rod at 77 and 87 K. The critical current, defined by the 1 μV/cm criterion at 77 K, is one order of magnitude larger than that at 87 K. The tests were conducted by gradually increasing the current density and were terminated when the heating effect became appreciable. It should be noted that the range of experimental data for current

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density at 87 K is relatively small when compared with that at 77 K, because the critical current density (defined by the 1-μV/cm criterion) is much smaller at 87 K than at 77 K. Dissipation and heating in the superconductor are much larger at 87 K than at 77 K. Therefore, it was necessary to terminate the experiment at 87 K at a relatively low current density.

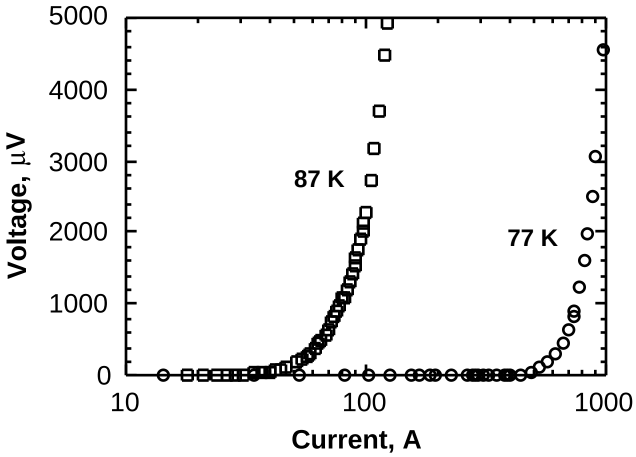

Fig. 1. Measured V/I characteristics at 77 and 87 K for a melt-cast processed BSCCO-2212 superconductor.

III. Magnetic Relaxation

Magnetic relaxation (or thermally activated flux creep) was first studied in low-temperature superconductors. At relatively low temperatures (≈4 K), the effect of magnetic relaxation in low-temperature superconductors was usually very small and especially sensitive experimental techniques were required to detect it. Furthermore, the relatively small specific heat of a low-temperature superconductor causes the superconductor to heat up rapidly when dissipation occurs as a result of either increased current density or temperature. Very often the amount of power dissipated is enough to heat up the low-temperature superconductor and quickly drive it into either the flux-flow or normal state. Consequently, the phenomenon of flux creep in low-temperature superconductors is often masked by a sudden increase in temperature. The situation is quite different for high-T_c superconductors. At relatively high temperatures (≈77 K), the rate of magnetic relaxation is large (it is sometimes referred to as giant flux creep) and the range of operating current for flux creep is much larger than that of low-temperature superconductors. Furthermore, at 77 K, the specific heat of the high-T_c superconductors can be two orders of magnitude larger than that of low-temperature superconductors at 4 K. For example, the volumetric specific heat of Nb_3Sn at 6.2 K is 0.6 x 10^4 J/m^3-K, whereas that of BSCCO-2212 at 77 K is 1.2 x 10^6 J/m^3-K. Therefore, a high-T_c superconductor will heat up much more slowly than a low-temperature superconductor for a similar amount of dissipation. These factors make the detection of magnetic relaxation [7] and magnetic diffusion [1-3] much easier in high-T_c superconductors than in low-temperature superconductors.

High-T_c superconductors usually obey the so-called power law,

$$E/E_c = (J/J_c)^n , \qquad (1)$$

where both E_c and J_c can be functions of temperature, and the exponent n is a strong function of temperature and can vary widely for various high-T_c superconductors. The power law is the result of magnetic relaxation and can be explained in terms of the Anderson-Kim model for thermally activated flux creep [7]. The basic concept is that magnetic flux lines can move in and out of the pinning sites because of thermal fluctuation. When flux lines are moving at a velocity v in a perpendicular magnetic field B, an electric field E,

$$E = v B , \qquad (2)$$

is generated. It is generally assumed that the velocity v is given by the relationship

$$v = v_0 \exp[-U(J)/kT] , \qquad (3)$$

where k is the Boltzman constant and U(J) is a current-dependent activation energy for depinning, which vanishes at the critical current density J_c [7,9]. Equations 2 and 3 can be combined to give

$$E(J) = E_c \exp[-U(J)/kT] , \qquad (4)$$

with $E_c = B\ v_0$. If a logarithmic dependence of U on J is assumed,

$$U(J) = U_0 \ln(J_c/J) , \qquad (5)$$

and the result is the power law shown in Eq. 1, with

$$n = U_0/kT . \qquad (6)$$

The parameter U_0 is independent of J, but it can be a function of T. From Eq. 5, it can be easily verified that, as $J = J_c$, $U = 0$, and consequently flux lines can move freely out of the pinning sites.

IV. Development of an Empirical Correlation for E(J,T)

Theory of flux creep or magnetic relaxation is of little help in determining the temperature dependence of U_0. However, we may get a hint from the argument that, when T approaches T_c, the superconductor becomes normal and turns into an Ohmic conductor. The exponent n should approach one and the E/J relationship becomes linear. The following equation satisfies this condition.

$$n = U_0/kT = 1 + C_0 \ln(T_c/T) , \qquad (7)$$

where C_0 is a constant. It is easily shown that n = 1 when $T = T_c$. From the experimental data at 77 and 87 K, it was found that C_0 40. Equation 1 can be written as

$$E(J,T) = f(T) \times J^n = f(T) \times J^{[1 + 40 \ln(T_c/T)]} , \qquad (8)$$

where the function f depends on temperature only and is equal to

$$f(T) = E_c / (J_c)^n . \quad (9)$$

Figure 2 shows the calculated E(J,T)/f(T) as a function of current density and temperature. It can be seen that E(J,T)/f(T) changes by one to two orders of magnitude when the temperature is changed by one degree and the change is fairly uniform. A function that satisfies these characteristics is

$$f(T) = A(T) \times 10^{2(T-89)} , \quad (10)$$

where A(T) is a coefficient to be determined by experimental data. A(T) can be a linear or a quadratic function of T. To ensure that E(J,T) remains positive to 92 K, we employed a quadratic function for A(T),

$$A(T) = a + bT + cT^2 . \quad (11)$$

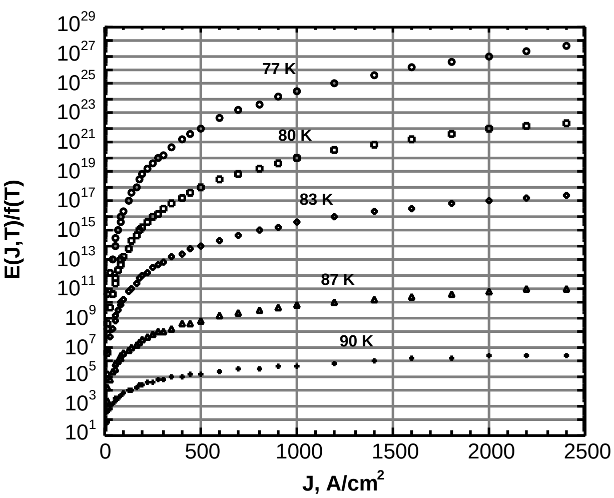

Fig. 2. Variation of E(J,T)/f(T) as a function of J for various temperatures (Eq. 8).

We know that the normal-state resistivity of the BSCCO-2212 superconductor at 92 K is ≈500 μΩ-cm. To utilize this condition, we take the derivative of E(J,T) with respect to the current density, and, from Eq. 8, we obtain

$$\rho(J,T) = \partial E(J,T)/\partial J = f(T) \times [1 + 40 \ln(T_c/T)] \times J^{[40 \ln(T_c/T)]}, \quad (12)$$

where ρ(J,T) is the resistivity of the superconductor. At T = T_c 92 K, we have

$$\rho(T_c) = 500 = f(T_c) = (a + bT_c + cT_c^2) \times 10^{2(T_c-89)} . \quad (13)$$

Equation 13 and the experimental data of E/J characteristics at 77 and 87 K (Fig. 1) can be used to determine the constants a, b, and c. It was determined that

$$a = 39.236680,\ b = -0.856427,\ \text{and}\ c = 0.004673 . \quad (14)$$

Thus, the final correlation for E(J,T) is

$$E(J,T) = (39.236680 - 0.856427T + 0.004673T^2) \times 10^{2(T-89)} \times J^{[1 + 40 \ln(T_c/T)]} , \quad (15)$$

where the temperature T is in Kelvin, the current density J is in A/cm^2, and the electrical field strength E is in μV/cm. Figure 3 shows the comparison of the E(J,T) calculated from Eq. 15, and the experimental data obtained at 77 and 87 K. The correlation appears to match the experimental data at 77 and 87 K fairly well and is capable of predicting the general trend of E as a function of current density at other temperatures between 77 and 92 K. The correlation developed here is specifically for the melt-cast processed BSCCO-2212 superconductor, because the data used to determine the coefficients are from Fig. 1. However, the general form of E(J,T) given by Eqs. 8-11 may be applicable to other bulk high-T_c superconductors.

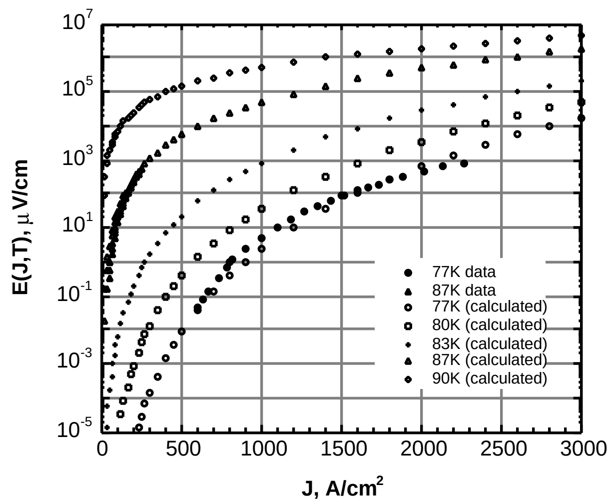

Fig. 3. Calculated E(J,T) and experimental data for a melt-cast processed BSCCO-2212 rod at 77 and 87 K. E(J,T) is calculated from Eq. 15.

It is usually the resistivity ρ(J,T) that is needed in the calculation of the temperature and magnetic flux density distribution during a transient [1]. The final correlation for the resistivity, from Eq. 12, is

$$\rho(J,T) = (39.236680 - 0.856427T + 0.004673T^2) \times 10^{2(T-89)} \times [1 + 40 \ln(T_c/T)] \times J^{[40 \ln(T_c/T)]} , \quad (16)$$

where the unit for the resistivity ρ in Eq. 16 is in μΩ-cm. Figure 4 shows the calculated resistivity as a function of the temperature and current density.

V. Flux-Flow Resistivity

The correlation developed previously is applicable to the superconductor in the flux-creep regime. If the current density is above some specific value $J_{ff}(T)$, the superconductor will be driven into the flux-flow state. In the flux-flow regime, the Bardeen-Stephen model for flux-flow resistivity can be employed. Thus, for $J > J_{ff}$,

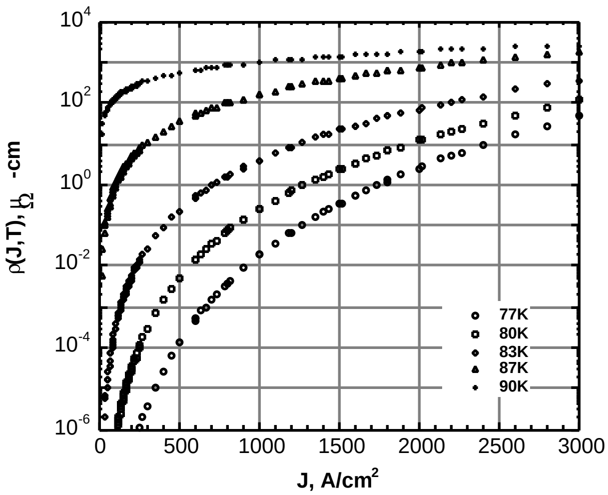

Fig. 4. Calculated resistivity $\rho(J,T)$ as a function of temperature and current density. $\rho(J,T)$ is calculated from Eq. 16.

$$\rho_{ff}(B,T) = \rho_0 \, [B / \mu_0 H_{c2}(T)] \, , \tag{17}$$

where B is the operating magnetic flux density, $H_{c2}(T)$ is the upper critical field, μ_0 is the permeability in free space, and ρ_0 is the resistivity of the superconductor in normal state. The normal-state resistivity ρ_0 is constant and independent of temperature. Strictly speaking, the normal-state resistivity ρ_0 should be evaluated at the local temperature because it is the resistivity of the normal core of a vortex [10]. However, this $\rho_0(T)$ is not a quantity that can be measured easily. For engineering applications, ρ_0 can be assumed to be equal to the measured normal-state resistivity at critical temperature $\rho_0(T_c)$. The upper critical field H_{c2} is a function of temperature and is usually determined from the phase diagram (H vs. T) of the superconductor. Near the critical temperature, H_{c2} decreases linearly with increasing temperature [10].

If dissipation and heating in the superconductor is small and cooling is sufficient to keep the superconductor at constant temperature, the resistivity will be independent of temperature. In this case, Brandt [11] proposed that the following expression be used to evaluate the resistivity:

$$\rho(J,B) = \rho_{ff}\{(J/J_c)^{n-1}/[1+(J/J_c)^{n-1}]\} \, . \tag{18}$$

Equation 18 combines the power law of the flux creep regime and ρ_{ff} of the flux flow regime. It can be easily shown that ρ approaches ρ_{ff} when $J >> J_c$ (flux flow), and $\rho \quad J^{n-1}$ when $J << J_c$ (power law for flux creep). Equation 18 is intended to be applicable only to isothermal systems. However, careful examination reveals that it might be applicable to nonisothermal systems if one considers that the flux-flow resistivity ρ_{ff}, the critical current density J_c, and the exponent n, are functions of temperature. Most likely all three parameters, ρ_{ff}, J_c, and n in Eq. 18, are nonlinear functions of temperature. The validity of using Eq. 18 for nonisothermal systems remains to be determined.

VI. Summary

We have developed an empirical correlation for E(J,T) of a melt-cast processed BSCCO-2212 superconductor based on experimental data at 77 and 87 K. The correlation (Eq. 15) is valid from 77 to 92 K, which is the range of interest for practical devices such as the high-T_c superconducting fault current limiters. The resistivity, which is usually needed to calculate the temperature and flux density distribution, is derived from the correlation for E(J,T) and is given by Eq. 16. Although the E(J,T) correlation is developed specifically for BSCCO-2212, the general form of the correlation (Eqs. 8-11) may be applicable to other high-T_c superconductors as well because the correlation is based, in part, on the general theory of magnetic relaxation (thermally activated flux creep).

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