Flow of a Two-Dimensional Liquid Metal Jet in a Strong Magnetic Field

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Abstract

Two-dimensional, steady flow of a liquid metal slender jet pouring from a nozzle in the presence of a transverse, nonuniform magnetic field is studied. The surface tension has been neglected, while gravity is shown to be not important. The main aim of the study is to evaluate the importance of the inertial effects. It has been shown that for gradually varying fields characteristic for the divertor region of a tokamak, inertial effects are negligible for $N > 10$, where N is the interaction parameter. Thus the inertialess flow model is expected to give good results even for relatively low magnetic fields and high jet velocity. Simple relations for the jet thickness and velocity have been derived. The results show that the jet becomes thicker if the field increases along the flow and thinner if it decreases.

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1. Introduction

Liquid metal free-surface flows offer the potential to solve the lifetime issues limiting solid surface designs for tokamak reactors by eliminating the problems of erosion and thermal stresses [1], [2]. They also provide the possibility of absorbing impurities and possibly helium for removal outside of the plasma chamber. In the US ALPS divertor concept this role is fulfilled by a curtain of liquid metal jets [1].

One of the most important problems for the liquid-metal divertors is the magnetohydrodynamic (MHD) interaction. When a liquid metal flows in a magnetic field, electric currents are induced. These currents in turn interact with the magnetic field and the resulting electromagnetic force induces a high MHD pressure drop and significant nonuniformities of the velocity profile. Although some experimental and theoretical work on the MHD jet flows has been performed, many issues still need to be resolved (see a review in [3]). Among the most important ones are the effects of nonuniform magnetic fields, inertia, surface tension, and gravity on both the jet cross-section and trajectory.

The main aim of this paper is to study inertial effects in a steady jet flow in a nonuniform magnetic field. As an initial step two-dimensional flow of a slender jet in the presence of a nonuniform, transverse magnetic field is considered. Surface tension effects are neglected, while the effect of gravity is shown to be negligible. The assumption of two-dimensionality means that the flow is confined laterally by two perfectly conducting sidewalls. Away from the immediate vicinity of the sidewalls the flow is two-dimensional (cf. [4]).

It should be emphasized that the jets in the divertor have a finite cross-section, and that the walls are either insulating or have a finite conductivity, so that this particular model flow has its limitations in terms of direct applicability to real divertor flows. In such flows inertial effects are difficult to analyse. In contrast, the geometry studied here presents an opportunity to get an assessment of the importance of inertial effects and to derive simple relations for the jet velocity and thickness, which may be used to verify numerical methods being developed for more realistic geometries.

2. Formulation

Consider a steady flow of a viscous, electrically conducting, incompressible fluid in a jet pouring downward in the x^*-direction (the direction of gravity) from a nozzle (Fig. 1). Here (x^*, y^*, z^*) is the Cartesian co-ordinate system. Dimensional quantities are denoted by letters with asterisks, while their dimensionless counterparts - with the same letters, but without the asterisks. For $x^* < 0$ the flow is between two parallel electrically insulating plates located at $y^* = \pm a^* / 2$, while for $x^* > 0$ there is a free-surface flow. The location of the free surface is defined as follows: $y^* = \pm h^*(x^*)$. The flow is supposed to be symmetric with respect to y^*, and therefore flow for $y^* > 0$ only will be considered.

The flow occurs in the presence of a strong, transverse magnetic field $\mathbf{B}^* = B_0^* B(2x^* / a^*)\hat{\mathbf{y}}$, where B_0^* is the induction of the uniform magnetic field in the upstream region. The magnetic field is supposed to be uniform ($\mathbf{B}^* = B_0^* \hat{\mathbf{y}}$) within the duct region, and nonuniform within the jet region. This is not an essential assumption but rather a matter of convenience only.

Laterally the flow is confined by perfectly conducting sidewalls located at $z^* = \pm L^*$, which are connected through a resistor, so that the resulting electric field E^* is supposed to be given (see the discussion below). In this case, sufficiently far from the sidewalls the electric current flows in the z^*-direction only, while the flow may be considered two-dimensional, in the (x^*, y^*)-plane [4].

The characteristic values of the length, the fluid velocity, the electric current density, the electric field and the pressure are $a^*/2$, $v_0^* = Q^*/a^*$ (average velocity in the duct region), $\sigma v_0^* B_0^*$, $v_0^* B_0^*$ and $a^* \sigma v_0^* B_0^{*2}$, respectively. In the above, σ, ρ, ν are the electrical conductivity, density and kinematic viscosity of the fluid, Q^* is the flow rate. Then the steady, dimensionless, two-dimensional, inductionless equations governing the flow are [3]:

$$j_z = E + uB(x)\,, \quad Ha^{-2}\nabla^2 u - j_z B(x) = \frac{\partial p}{\partial x} - \delta + N^{-1}\left\{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right\}, \qquad \text{(1a,b)}$$

$$Ha^{-2}\nabla^2 v = \frac{\partial p}{\partial y} + N^{-1}\left\{u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right\}, \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \qquad \text{(1c,d)}$$

where u and v are the x- and y- components of the fluid velocity, respectively, p is the pressure, j_z is the z-component of current, $Ha = a^* B_0^* (\sigma/\rho\nu)^{1/2}$ is the Hartmann number, which expresses the ratio of the electromagnetic to the viscous force, $N = a^* \sigma B_0^{*2}/\rho v_0^*$ is the interaction parameter, which expresses the ratio of the electromagnetic to the inertial force, $\delta = \rho g/\sigma v_0^* B_0^{*2}$ is the parameter, which expresses the ratio of the gravitational to the electromagnetic force. In fact, parameter δ equals $(FrN)^{-1}$ [3], where $Fr = v_0^{*2}/a^* g$ is the Froude number. Typical values of dimensionless parameters are given in Table 1, from which follows that δ is much lower than other terms in the momentum equation, and thus in the following gravity is neglected.

The symmetry conditions are:

$$\partial u/\partial y = 0\,, \quad v = 0 \quad \text{at } y = 0. \qquad \text{(1e,f)}$$

The boundary conditions at the duct wall are the no-slip- conditions:

$$u = v = 0 \quad \text{at } y = 1. \qquad \text{(1g,h)}$$

The boundary conditions at the free surface are the kinematic and the dynamic boundary conditions:

$$v = uh', \;\; [S]\cdot\hat{\mathbf{n}} = 0 \quad \text{at } y = h(x), \qquad \text{(1i,j)}$$

where $[S]$ is the jump of the stress tensor across the free surface, $\hat{\mathbf{n}} = (h'\hat{\mathbf{x}} - \hat{\mathbf{y}})/(1+h'^2)^{1/2}$ is the normal unit vector to the free surface pointing into the fluid (Fig. 1), and ‘ = *d/dx*. In Eq. (1j) surface tension term has been neglected.

Far upstream the flow is fully developed, which requires

$$\partial p/\partial x \to \text{constant}, \quad v \to 0 \quad \text{as } x \to -\infty. \qquad \text{(1k,}l\text{)}$$

Finally, the solution is normalized using the condition of a fixed flow rate:

$$\int_0^1 u\,dy = 1 \quad \text{for } x < 0, \quad \text{and} \quad \int_0^{h(x)} u\,dy = 1 \quad \text{for } x > 0. \qquad \text{(1m,n)}$$

3. High-*Ha* flow model

In this section the problem defined by Eqs. (1) is analysed for high values of *Ha* and for sufficiently high values of *N*. The restriction on *N* is determined by the requirement that the flow remains laminar. Since no experimental data for transition to turbulence in a jet exist, we base the restriction on *N* on the available data for the duct flow. Therefore, we require that $N/Ha > 0.004$ [5]. As follows from Table 1 this condition holds for all the three values of the field, and therefore the flow is expected to be laminar.

In the following all the flow variables denote their core values. Terms $O(Ha^{-1})$ are neglected.

In a sufficiently strong magnetic field the flow region splits into the following subregions (Fig. 1): the cores *C1*, *C2*, the Hartmann layers *H1*, *H2* of thickness $O(Ha^{-1})$ at the wall and the free surface, respectively, and the internal parallel layer *S* at $x = 0$ of thickness $O(Ha^{-1/2})$. The analysis shows that layer *S* is passive, and is not considered here. There are also corner layers at $x = 0$, $y = \pm 1$ (also being passive; not shown in the Fig. 1) with dimensions $O(Ha^{-1}) \times O(Ha^{-1})$.

The Hartmann layers *H1*, *H2* provide matching conditions for the variables in cores *C1* and *C2*. In the duct region (*C1*) this is the non-penetration condition (1h). In the jet region the Hartmann layer *H2* vanishes to $O(1)$. As a result for the core *C2* the kinematic condition (1i) holds, while the dynamic condition (1j) reduces to

$$p = 0 \quad \text{at } y = h(x). \tag{2}$$

The constant pressure of the surrounding medium is set to zero.

3.1 Cores C1, C2

The analysis for both cores goes along the same lines. In the cores the flow is inviscid. The inertial term $N^{-1} u \partial u / \partial x$ in Eq. (1b) is retained, while all other inertial terms in Eqs. (1b,c) are neglected. This requires that the jet is slender ($h' \ll 1$) (see Appendix in [6]). Substituting Eq. (1a) into the truncated version of Eq. (1b) and using Eqs. (1c,d,f) yields:

$$p'(x) = -[E + u(x)B(x)]B(x) - N^{-1} u(x) u'(x), \quad v = y u'(x), \tag{3a,b}$$

where both *u* and *p* are functions of *x* only.

In the *duct region* the non-penetration condition (1h) holds. Then Eqs. (3a,b) and (1m) give the solution for the core *C1* as follows:

$$v = 0, \quad u = 1, \quad p = -(E+1)x, \tag{4a-c}$$

i.e. the Hartmann profile holds up to the junction. It should be noted that for E = -1 the pressure gradient is $O(Ha^{-1})$ and is neglected since we are interested in $O(1)$ terms only.

For the *jet region* applying the kinematic condition (1i) to the core variables and using Eq. (1n) gives:

$$u(x) = h^{-1}(x). \tag{5}$$

Applying condition (2) yields p = 0 in the whole jet region, while Eqs. (3a) and (5) give the following first-order nonlinear differential equation for h:

$$h' = Nh^2 B(x)\left[Eh + B(x)\right] \quad \text{for } x > 0. \tag{6}$$

This equation is to be solved numerically using the boundary condition $h(0) = 1$.

4. Results

4.1 Flow in a uniform field (B = 1)

If the field is uniform, Eq. (6) yields a solution for the jet in an implicit form as follows:

$$x = N^{-1}\left\{E \ln\left|(E + h^{-1})/(E+1)\right| + 1 - h^{-1}\right\} \quad \text{for } x > 0. \tag{7}$$

The interaction parameter N simply scales the x-co-ordinate.

Variation of the jet thickness with xN for several values of E is shown in Fig. 2.

The case E = -1, which approximates duct/jet flows of finite cross-section best, corresponds to infinite resistance between the sidewalls. Indeed, as follows from Eqs. (1a,m) and (5), for each x the total current through the duct cross-section equals to zero. Then from Eq. (7) follows that h = 1, so that the electric field fully compensates induced electromotive force (e.m.f.) in both the duct and jet regions, and there is no net electric current in the jet either.

As E increases from –1 to 0, the current in the jet region becomes non-zero, producing a Lorentz force opposing the fluid velocity. This Lorentz force is balanced by inertia. For all cases presented in Fig. 2, except for E = 0, inertia acts over a distance of about $2N^{-1}$, which is less than one value of the characteristic length for all the three values of the magnetic field presented in Table 1.

For E = 0 (sidewalls shortcut) the solution can be obtained in an explicit form:

$$h(x) = [1 - Nx]^{-1}, \quad u(x) = 1 - Nx. \qquad (8a,b)$$

This means that the jet is completely diverted in the ±y-direction at a distance $x = N^{-1}$. The reason for this is that there is no electric field to compensate the e.m.f. This is similar to the result for a submerged two-dimensional jet studied in [7]. As N tends to infinity, the flow becomes inertialess, while the jet will be diverted in the ±y-direction right at the junction, in the S-layer.

The physical significance of the flow for E = 0 is the following. Consider an axisymmetric curtain with no dividing walls falling down vertically along the circumference of a tokamak. Then the geometry studied here approximates a radial-poloidal cross-section of such a curtain in a horizontal, transverse radial/poloidal field. Since the flow is axisymmetric, the electric field vanishes identically, and thus the solution presented above applies.

For E < -1, the electric field accelerates the jet, which becomes thinner.

4.2 Flow in a nonuniform field for E = -1

In the following the flow in a nonuniform magnetic field for E = -1 will be discussed. Consider first the inertialess flow. As $N \to \infty$ Eqs. (5) and (7) give:

$$h(x) = B(x), \quad u(x) = B^{-1}(x), \qquad (9a,b)$$

i.e. the jet thickness equals *B*. Therefore, the jet becomes thicker if the magnetic field increases along the flow and thinner if the field decreases.

For a finite *N* Eq. (7) is solved numerically. The results are presented in Fig. 3 for $B = 1 + (B_\infty - 1)\tanh\gamma x$, where B_∞ is the induction of the uniform magnetic field as $x \to \infty$, and γ is the gradient of the magnetic field equal to 0.2. The small value of γ has been selected to reflect gradual variation of the magnetic field in the divertor area, over a distance of about 10 values of the characteristic length [8]. The results show that for both N = 334 and N = 83.5 the jet thickness is practically indistinguishable from the inertialess one, given by Eq. (9a). This result is valid for $N > 10$ for all the cases presented in Fig. 3. The difference between the cases N = 3.34 and $N = \propto$ is significant only for the field dropping by a factor of 4 in the downstream region (curves 1, 2 and 3).

5. Conclusions

Two-dimensional, steady flow of a liquid metal slender jet pouring from a nozzle in the presence of a transverse magnetic field has been studied. The surface tension has been neglected.

It has been shown that gravity is negligible, while inertial effects are negligible for $N > 10$. The thickness of the inertialess jet may be approximated by a simple expression $h = B$. The jet becomes thicker if the field increases along the flow and thinner if it decreases.

The work on flows of a jet with a finite cross-section, including surface tension and unsteady effects is in progress.

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Table 1. Typical values of parameters for Li divertors for $a^* = 0.005m$, $v_0^* = 10m/s$. Thermophysical data for Li have been taken from [3].

B_0^*	10T	5T	1T
Ha	4308	2154	431
N	334	83.5	3.34
N/Ha	0.078	0.039	0.0078
Fr	2039	2039	2039
δ	$1.47 \cdot 10^{-6}$	$5.87 \cdot 10^{-6}$	$1.47 \cdot 10^{-4}$

Figure captions

Fig. 1 Schematic diagram of the jet flow and flow subregions for high *Ha*.

Fig. 2 Variation of h with xN for different values of E and for $B = 1$.

Fig. 3 Variation of h with x for $E = -1$ and for various values of N and B_∞: $B_\infty = 0.25$, $N = 334$ (1), $B_\infty = 0.25$, $N = 83.5$ (2), $B_\infty = 0.25$, $N = 3.34$ (3), $B_\infty = 0.5$, $N = 334$ (4), $B_\infty = 0.5$, $N = 83.5$ (5), $B_\infty = 0.5$, $N = 3.34$ (6), $B_\infty = 1.25$, $N = 334$ (7), $B_\infty = 1.25$, $N = 83.5$ (8), $B_\infty = 1.25$, $N = 3.34$ (9), $B_\infty = 2$, $N = 334$ (10), $B_\infty = 2$, $N = 83.5$ (11), $B_\infty = 2$, $N = 3.34$ (12).

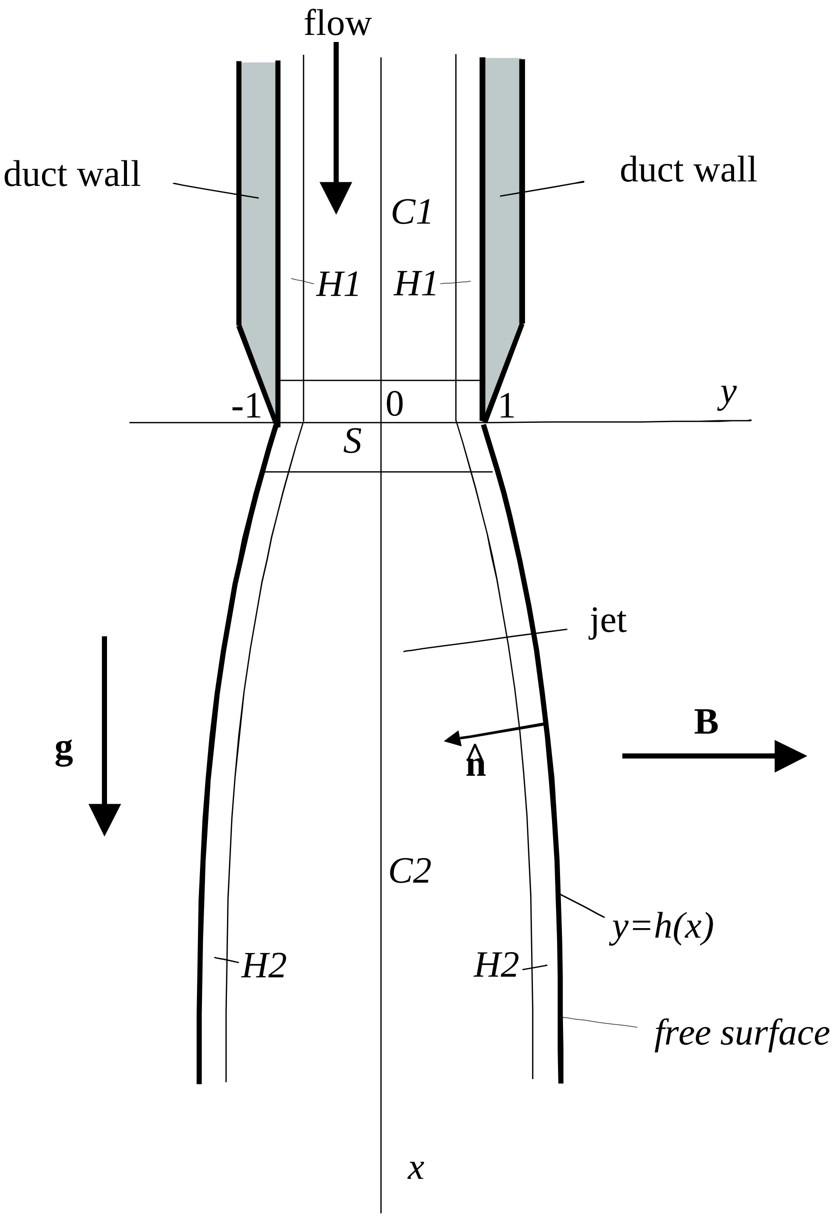

Fig. 1
Molokov and Reed. Flow of a two-dimensional jet

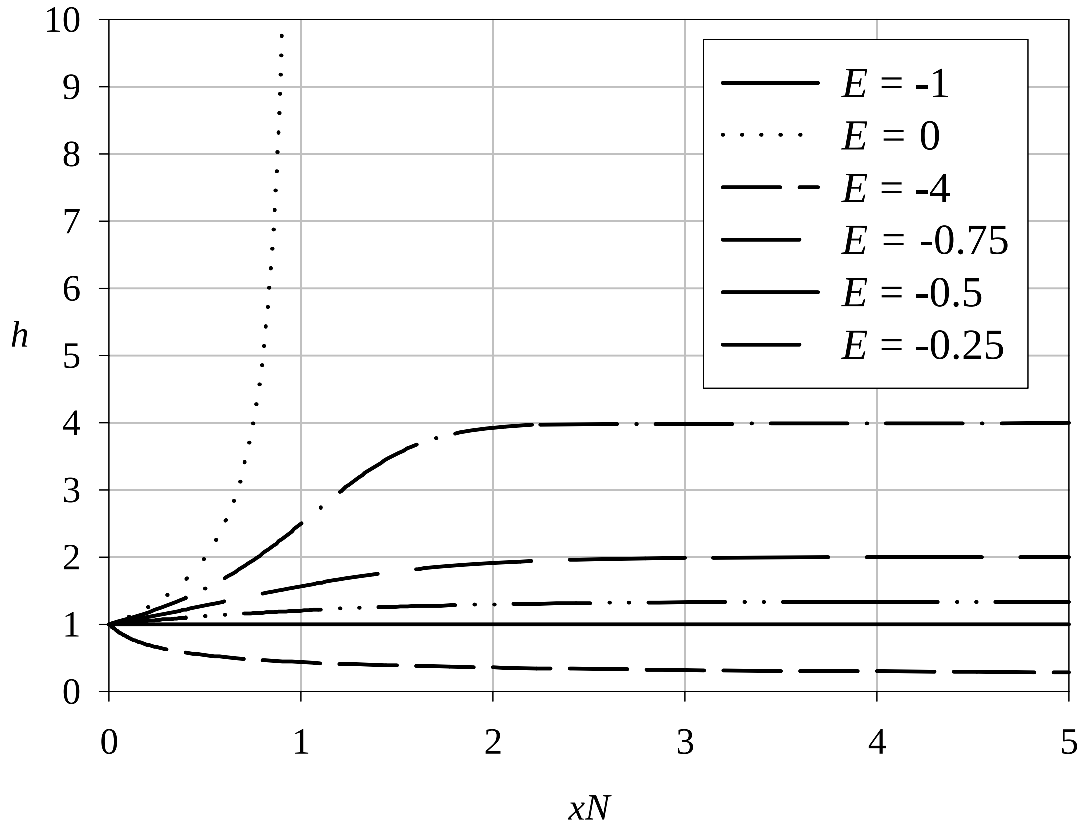

Fig. 2

Molokov and Reed. Flow of a two-dimensional jet

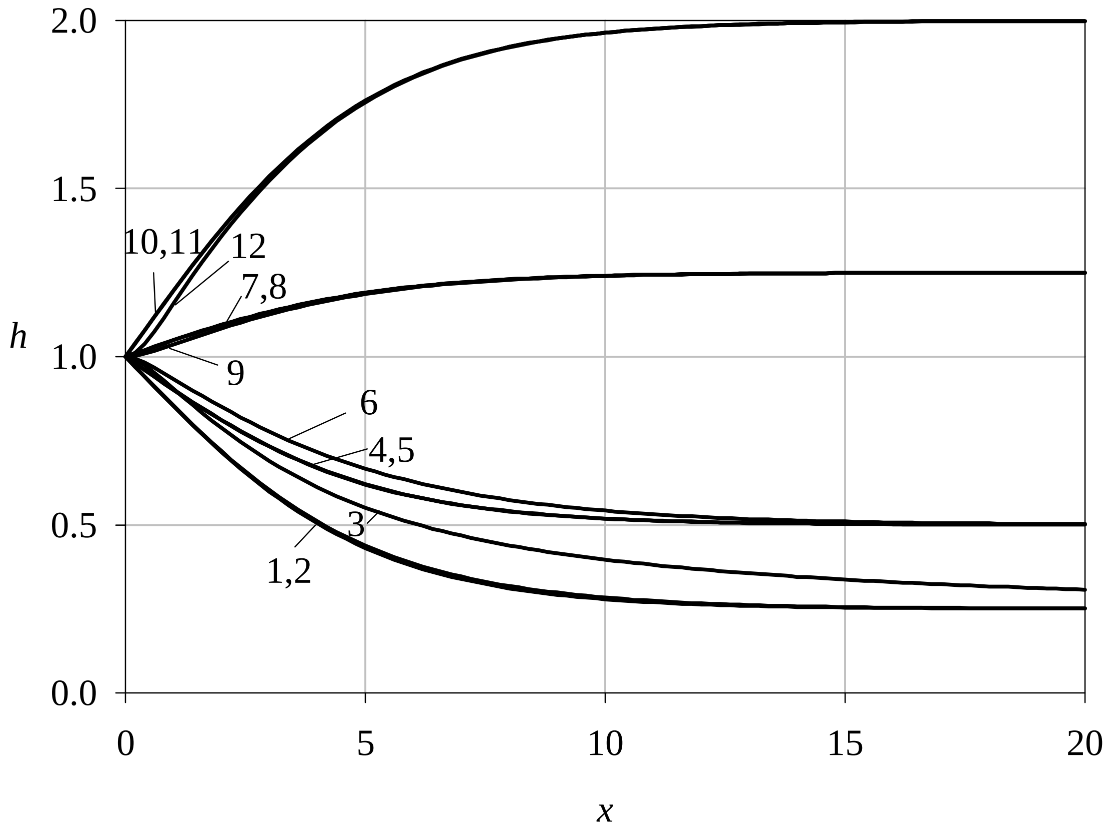

Fig. 3

Molokov and Reed. Flow of a two-dimensional jet