New Theoretical Results on the Proton Decay of Deformed and Near-Spherical Nuclei*)

Cary N. Davids and Henning Esbensen

Physics Division, Argonne National Laboratory, Argonne, IL 60439

(Received)

We discuss new theoretical results on the decay of deformed and near-spherical nuclei. We interpret the latest experimental results on deformed odd-A proton emitters, including fine structure, and discuss the use of particle-vibration coupling to calculate the decay rates of near-spherical emitters.

§1. Introduction

Ground-state proton radioactivity is a phenomenon associated with nuclides lying beyond the proton drip line, and the study of this process provides nuclear structure information on nuclides very far from the line of beta stability [1]. The experimentally measured quantities are the proton decay energies and decay rates. Current models of proton radioactivity start with the assumption that the parent nucleus can be described as a single proton interacting with a core nucleus via a single-particle potential. This potential includes Coulomb, nuclear, and spin-orbit terms. Instead of treating the problem as a time-dependent one, a static solution of the Schrödinger equation is sought, but with outgoing Coulomb wave boundary conditions. This produces a solution with the correct properties at infinite distance. At the present time, decay rate calculations for deformed and near-spherical emitters are treated separately, due to the fact that deformed emitters are more easily treated in the body-fixed system, while near-spherical emitters can be handled in the laboratory system.

§2. Deformed Proton Emitters

Much progress in calculating the decay rates of deformed proton emitters has been achieved in recent years. Using the coupled channels approach, a number of studies have been performed, both in the adiabatic limit[2)–5)] and in the non-adiabatic case[4),5)]. In general, better agreement with experiment is achieved using the adiabatic limit [5)]. This is due to the presence of Coriolis coupling in the non-adiabatic case, which has a large effect on the decay rates of the high-spin proton emitters. Coriolis coupling is expected to be reduced to a more reasonable level when the effects of pairing are taken into account, but so far this has not been done in a consistent fashion.

*) Supported by the U.S. Department of Energy, Nuclear Physics Division, under Contract W-31-109-ENG-38.

The picture we consider here is that of a deformed odd-A nucleus, consisting of a proton strongly coupled to a reflection- and axially-symmetric even-even core. As a result, the total angular momentum j of the proton is no longer a good quantum number, and only the parity and K, the projection of j on the symmetry axis, are good quantum numbers. We proceed by using deformed Coulomb and nuclear potentials, and expanding the wavefunction in spherical components in the intrinsic system:

$$\psi_K(\mathbf{r}) = \sum_{\ell j} \frac{\phi_{\ell j}(r)}{r} |\ell j K\rangle, \tag{2·1}$$

where the sum is over $j \geq |K|$, with fixed parity. We then insert (2·1) into the Schrödinger equation $H\psi_K(\mathbf{r}) = E\psi_K(\mathbf{r})$. After projecting this equation with the spin-angular part of the wavefunction $|\ell j K\rangle$ we obtain a set of coupled differential equations for the radial wavefunctions $\phi_{\ell j}(r)$:

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2} - E\right]\phi_{\ell j}(r) = -\sum_{\ell' j'} \langle \ell j K | V_{def}(\mathbf{r}) | \ell' j' K\rangle \phi_{\ell' j'}(r). \tag{2·2}$$

Next we make a multipole expansion of the total deformed interaction, $V_{def}(\mathbf{r})$, consisting of Coulomb, nuclear, and spin-orbit potentials:

$$V_{def}(\mathbf{r}) = \sum_{\lambda=0} V_\lambda(r) P_\lambda(cos(\theta')), \tag{2·3}$$

where θ' is the angle with respect to the symmetry axis. Only even values of λ appear in the summation because of the reflection- and axial symmetry.

The (real) resonance solutions to the coupled equations (2·2) are obtained by matching each $\phi_{\ell j}(r)$ to the irregular Coulomb function $G_\ell(kR)$. In ref. [5] this is done at a relatively small distance from the nucleus, 15 fm. The distorted wave Green's function method [6] is used to calculate the decay width, taking into account the differences between intrinsic and laboratory frames. For a given angular momentum ℓ_p carried off by the proton, the decay width of an odd-A proton emitter with spin I and projection K ($I = K$), where the daughter nucleus with atomic number Z is left in a state with spin R_d, is given by

$$\Gamma^{R_d I K}_{\ell_p j_p} = \frac{4\mu}{\hbar^2 k}\frac{2(2R_d+1)}{(2I+1)} \langle j_p K R_d 0 | I K\rangle^2 \left|\mathcal{M}^K_{\ell_p j_p}\right|^2,$$

where

$$\mathcal{M}^K_{\ell_p j_p} = \sum_{\lambda \ell j} \langle \ell_p j_p K | P_\lambda(cos(\theta')) | \ell j K\rangle \int_0^R dr F_{\ell_p}(kr) \left[V_\lambda(r) - \delta_{\lambda 0}\frac{Ze^2}{r}\right] \phi_{\ell j}(r).$$

The wavenumber k of the proton is determined from the decay Q-value Q_p, atomic screening correction E_{sc}, and excitation energy E_x of the daughter state. By carrying out the integration to a radius $R = 100$ fm, the effect of the long-range Coulomb field is taken into account quite accurately.

Once the decay width has been calculated, it is compared with experiment by calculating the spectroscopic factor, defined as the ratio between experimental and calculated decay widths. The spectroscopic factor represents the degree of overlap between the proton plus daughter state with the parent proton emitter. This quantity is normally less than unity due to pairing, which has not been considered here. It would tend to reduce the decay width by 30-50%.

§3. Fine Structure in Proton Decay

Recently, fine structure in proton decay has been observed in the deformed proton emitter ^{131}Eu [7]. For an odd-A proton emitter in the adiabatic limit, only the wavefunction component of the parent state with $j = K$ can decay to the 0^+ ground state of the daughter. However, decay to the 2^+ state can have contributions from wavefunction components with $K \leq j \leq K + 2$. If the ground-state-to-ground state decay is relatively weak and a large wavefunction component with the same ℓ-value is involved in the decay to the 2^+ state, a significant 2^+ branching ratio can be obtained. This is the case for the deformed emitter ^{131}Eu, where a 2^+ branching ratio of 24(5)% was obtained [7].

Using the formalism described above, we have calculated the decay widths for the deformed proton emitters $^{131}\mathrm{Eu}(\frac{3}{2}^+)$, $^{141g}\mathrm{Ho}(\frac{7}{2}^-)$, and $^{141m}\mathrm{Ho}(\frac{1}{2}^+)$, as well as the branching ratios for the decays to the 2^+ states of the daughter nuclei ^{130}Sm and ^{140}Dy, respectively. The results are shown in Table I. The calculated branching ratios are in good agreement with experiment, and pairing may very well account for the spectroscopic factors. An exception is $^{141m}\mathrm{Ho}(\frac{1}{2}^+)$, where poor overlap of the core and the daughter nucleus may be responsible for the small spectroscopic factor.

Table I. Calculated and experimental proton decay widths (in units of 10^{-20} MeV). The 2^+ widths Γ_2 have been calculated with $E_{2^+} = 122$ keV for ^{130}Sm and $E_{2^+} = 202$ keV [8] for ^{140}Dy. S_{exp} is the experimental spectroscopic factor. Pairing is not included.

Nucleus	Γ_0	S_{exp}	Γ_2	$\frac{\Gamma_2}{(\Gamma_0+\Gamma_2)}$
$^{131}\mathrm{Eu}(3/2^+)$	2.88	-	0.929	0.244
Experiment [7]	1.71(24)	0.59	0.54(13)	0.24(5)
$^{141g}\mathrm{Ho}(7/2^-)$	15.0	-	0.10	0.007
Experiment [9]	10.9(10)	0.73	-	< 0.01
$^{141m}\mathrm{Ho}(1/2^+)$	24800	-	94.5	0.004
Experiment [10]	5700(2140)	0.23	-	-
Experiment [9]	7020(1080)	0.28	-	< 0.01

§4. Near-Spherical Proton Emitters

The recent observation of fine structure in the proton decay of ^{145}Tm [11] means that the simple spherical approach does not provide a full description of the proton emission process for near-spherical nuclides. Two proton groups were observed with the same half-life of 3.0(3) μs, one populating the ground state and the other populating the first 2^+ state of the daughter ^{144}Er at 0.33 MeV with a branching ratio of 9.6(15)% [11]. This suggests that ^{145}Tm and other near-spherical proton emitters may have a time-averaged spherical shape, but their wavefunctions contain other components. The even-even daughters of the odd-A proton emitters between $^{145}_{69}$Tm and $^{177}_{81}$Tl have a ratio of excitation energies $E_x(4^+)/E_x(2^+)$ lying in the range 2.0-2.4, characteristic of an anharmonic vibrator. This indicates that the interaction between the last proton and the core nucleus should include particle-vibration coupling to the first 2^+ state of the daughter nucleus.

§5. Coupled-Channels Approach with Particle-Vibration Coupling

We have used the coupled-channels Green's function method [16] to calculate the proton decay rates of both odd-A and odd-odd near-spherical proton emitters whose even-even core nuclei display vibrational properties. Since we are dealing with nuclei having 68 < Z < 82, the $1h_{11/2}$, $2d_{3/2}$, $3s_{1/2}$, and higher shell model orbitals are available for inclusion in the coupled equations. As described in ref. [16], the strength of the vibrational coupling term has been determined from experimental values for the excitation energy of the first 2^+ state in the core. We have not included calculations for odd-A ^{155,157}Ta, whose daughter nuclei ^{154,156}Hf do not appear to be vibrational.

As already mentioned, fine structure has been observed in the decay of ^{145}Tm. Using particle-vibration coupling, the calculated half-life of 2.0 μs and 2^+ branching ratio of 9.9% are in remarkably good agreement with the experimental values [11], yielding a spectroscopic factor of 0.66(8). Fig. 1(a) shows the experimental spectroscopic factors S_{exp} for the odd-A proton emitters plotted as a function of p, the number of pairs of proton holes below Z=82 in the daughter nucleus. Excellent agreement is obtained with the low-seniority shell model calculation of spectroscopic factors described in Ref. [17], which is shown as the solid line in Fig. 1(a-c). In contrast to previous work [17],[18], the present value of S_{exp} for the odd-A $d_{3/2}$ proton emitter ^{147}Tm^m agrees extremely well with the calculated value. The single-particle potential parameters used are identical to those used to successfully describe the decay of the deformed proton emitters ^{131}Eu and ^{141}Ho [5]. This demonstrates for the first time that the same single-particle potential can be used to describe proton decay of both near-spherical and deformed nuclei.

Fig. 1(b) shows the experimental spectroscopic factors calculated in a spherical picture with no particle-vibration coupling for the same emitters as in Fig. 1(a). The agreement with the theoretical spectroscopic factors is now spoiled, clearly showing the important role played by particle-vibration coupling.

For the odd-odd emitters, shown in Fig. 1(c), only the decay width to the daughter ground state is presented here. The unpaired neutron is considered to be a spectator in these calculations, and the assumed core nucleus is the even-even nucleus (Z-1,A-2). Again, we have not included calculations for ^{156}Ta, whose core nucleus ^{154}Hf does not appear to be vibrational.

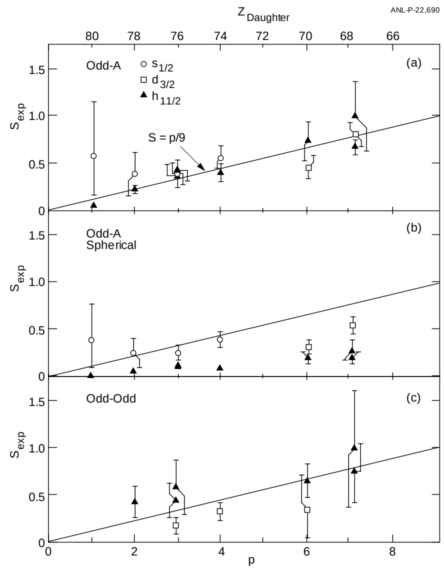

Fig. 1. (a) Experimental spectroscopic factors S_{exp} calculated with particle-vibration coupling for the odd-A proton emitters, plotted as a function of p, the number of pairs of proton holes below Z=82 possessed by the daughter nucleus. The atomic number of the daughter nucleus Z_D is also shown, with $Z_D = 82 - 2p$. (b) Experimental spectroscopic factors for spherical odd-A proton emitters (no particle-vibration coupling). (c) Same as (a) except for odd-odd proton emitters.

§6. Summary

Much progress has been made in calculating the decay rates for deformed and near-spherical proton emitters. The observation of fine structure in proton decay provides important clues on the wavefunction composition of proton emitters. Particle-vibration coupling is necessary to explain the decay rates for near-spherical emitters.

References

1) P. J. Woods and C. N. Davids, Annu. Rev. Part. Sci 47, 541 (1997).
2) V. P. Bugrov and S. G. Kadmensky, Phys. At. Nucl. **59**, 424 (1996) and references therein.
3) E. Maglione, L. S. Ferreira, Phys. Rev. C **61**, 047307 (2000) and references therein.
4) B. Barmore et al., Phys. Rev. C **62**, 054315 (2000).
5) Henning Esbensen and Cary N. Davids, Phys. Rev. C **63**, 014315 (2001).
6) Cary N. Davids and Henning Esbensen, Phys. Rev. C **61**, 054302 (2000).
7) A. A. Sonzogni et al., Phys. Rev. Lett. **83**, 1116 (1999).
8) M. P. Carpenter et al., submitted to Phys. Lett. B.
9) D. Seweryniak et al., Phys. Rev. Lett. **86**, 1458 (2001).
10) K. P. Rykaczewski et al., Phys. Rev. C **60**, 011301 (1999).
11) K. P. Rykaczewski et al., Nucl. Phys. A682, 270c (2001), and private communication.
12) L. S. Ferreira and E. Maglione, Phys. Rev. C **61**, 021304(R) (2000).
13) P. Möller et al., At. Data Nucl. Data Tables **59**, 185 (1995).
14) Y. Aboussir et al., At. Data Nucl. Data Tables **61**, 127 (1995).
15) G. Lalazissis, D. Vretenar, and P. Ring, Nucl. Phys. A 650, 133 (1999).
16) Cary N. Davids and Henning Esbensen, Phys. Rev. C **64**, 034317 (2001).
17) C. N. Davids et al., Phys. Rev. C **55**, 2255 (1997).
18) S. Åberg, P. B. Semmes, and W. Nazarewicz, Phys. Rev. C **56**, 1762 (1997).