New Theoretical Results on the Proton Decay of Deformed and Near-Spherical Nuclei*

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Abstract

We discuss new theoretical results on the decay of deformed and near-spherical nuclei. We interpret the latest experimental results on deformed odd-A proton emitters, including fine structure, and discuss the use of particle-vibration coupling to calculate the decay rates of near-spherical emitters.

1 Introduction

Ground-state proton radioactivity is a phenomenon associated with nuclides lying beyond the proton drip line, and the study of this process provides nuclear structure information on nuclides very far from the line of beta stability [1]. The experimentally measured quantities are the proton decay energies and decay rates. Current models of proton radioactivity start with the assumption that the parent nucleus can be described as a single proton interacting with a core nucleus via a single-particle potential. This potential includes Coulomb, nuclear, and spin-orbit terms. Instead of treating the problem as a time-dependent one, a static solution of the Schrödinger equation is sought, but with outgoing Coulomb wave boundary conditions. This produces a solution with the correct properties at infinite distance. At the

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present time, decay rate calculations for deformed and near-spherical emitters are treated separately, due to the fact that deformed emitters are more easily treated in the body-fixed system, while near-spherical emitters can be handled in the laboratory system.

2 Deformed Proton Emitters

Much progress in calculating the decay rates of deformed proton emitters has been achieved in recent years. Using the coupled channels approach, a number of studies have been performed, both in the adiabatic limit [2-5] and in the non-adiabatic case [4, 5]. In general, better agreement with experiment is achieved using the adiabatic limit [5]. This is due to the presence of Coriolis coupling in the non-adiabatic case, which has a large effect on the decay rates of the high-spin proton emitters. Coriolis coupling is expected to be reduced to a more reasonable level when the effects of pairing are taken into account, but so far this has not been done in a consistent fashion.

The picture we consider here is that of a deformed odd-A nucleus, consisting of a proton strongly coupled to an axially-symmetric even-even core. As a result, the total angular momentum j of the proton is no longer a good quantum number, and only the parity and K, the projection of j on the symmetry axis, are good quantum numbers. We proceed by using deformed Coulomb and nuclear potentials, and expanding the wavefunction in spherical components in the intrinsic system:

$$\psi_K(\mathbf{r}) = \sum_{\ell j} \frac{\phi_{\ell j}(r)}{r} |\ell j K\rangle, \tag{1}$$

where the sum is over $j \geq |K|$, with fixed parity. Here the spin-angular wavefunction $|\ell j K\rangle$ is shorthand for

$$|\ell j K\rangle = \sum_{m_\ell m_s} \langle \ell m_\ell \tfrac{1}{2} m_s | j K\rangle Y_\ell^{m_\ell}(\hat{\mathbf{r}}) \chi(m_s).$$

We then insert (1) into the Schrödinger equation $H\psi_K(\mathbf{r}) = E\psi_K(\mathbf{r})$. After projecting this equation with spin-angular part of the wavefunction $|\ell j K\rangle$ we obtain a set of coupled differential equations for the radial wavefunctions

$\phi_{\ell j}(r)$:

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{\hbar^2\ell(\ell+1)}{2\mu r^2}-E\right]\phi_{\ell j}(r)=-\sum_{\ell' j'}\langle \ell j K|V_{def}(\mathbf{r})|\ell' j' K\rangle\phi_{\ell' j'}(r). \quad (2)$$

Next we make a multipole expansion of the total deformed interaction, $V_{def}(\mathbf{r})$, consisting of Coulomb, nuclear, and spin-orbit potentials:

$$V_{def}(\mathbf{r})=\sum_{\lambda=0}V_\lambda(r)P_\lambda(cos(\theta')), \quad (3)$$

where θ' is the angle with respect to the symmetry axis. Only even values of λ appear in the summation because of the axial symmetry.

The (real) resonance solutions to the coupled equations (2) are obtained by matching each $\phi_{\ell j}(r)$ to the irregular Coulomb function $G_\ell(kR)$. In ref. [5] this is done at a relatively small distance from the nucleus, 15 fm. This significantly reduces the amount of computation required. The total wavefunction $\psi_K(\mathbf{r})$ is then normalized by requiring

$$\sum_{\ell j}\int_0^R[\phi_{\ell j}(r)]^2dr=1,$$

where $R\approx 100$ fm. The distorted wave Green's function method [6] is used to calculate the decay width, taking into account the differences between intrinsic and laboratory frames. For a given angular momentum ℓ_p carried off by the proton, the decay width of an odd-A proton emitter with spin I and projection K ($I=K$), where the daughter nucleus with atomic number Z is left in a state with spin R_d, is given by

$$\Gamma^{R_d IK}_{\ell_p j_p}=\frac{4\mu}{\hbar^2 k}\frac{2(2R_d+1)}{(2I+1)}\langle j_p K R_d 0|IK\rangle^2\left|\mathcal{M}^K_{\ell_p j_p}\right|^2,$$

where

$$\mathcal{M}^K_{\ell_p j_p}=\sum_{\lambda\ell j}\langle \ell_p j_p K|P_\lambda(cos(\theta'))|\ell j K\rangle\int_0^R drF_{\ell_p}(kr)\left[V_\lambda(r)-\delta_{\lambda 0}\frac{Ze^2}{r}\right]\phi_{\ell j}(r).$$

The wavenumber k of the proton is determined from the decay Q-value Q_p, atomic screening correction E_{sc}, and excitation energy E_x of the daughter state:

$$\frac{(\hbar k)^2}{2\mu}=Q_p-E_{sc}-E_x.$$

By carrying out the integration to a radius $R = 100$ fm, the effect of the long-range Coulomb field is taken into account.

Once the decay width has been calculated, it is compared with experiment by calculating the spectroscopic factor, defined as the ratio between experimental and calculated decay widths. The spectroscopic factor represents the degree of overlap between the proton plus daughter state with the parent proton emitter. This quantity is normally less than unity due to pairing, which has not been considered here. It would tend to reduce the decay width by 30-50%.

3 Fine Structure in Proton Decay

Recently, fine structure in proton decay has been observed in the deformed proton emitter ^{131}Eu [7]. Figure 1 shows the decay scheme of a nucleus that exhibits fine structure.

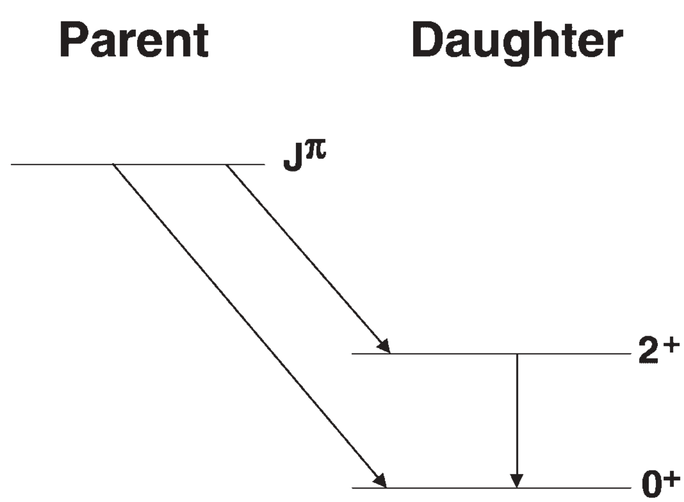

Figure 1: Illustration of fine structure in the proton decay of an odd-A nucleus. Proton branches take place between the parent state and both ground- and first excited state of the daughter nucleus. The excited state promptly decays by gamma emission or electron conversion.

For an odd-A proton emitter in the adiabatic limit, only the wavefunction component of the parent state with $j = K$ can decay to the 0^+ ground state of the daughter. However, decay to the 2^+ state can have contributions from wavefunction components with $K \leq j \leq K+2$. If the ground-state-to-ground state decay is relatively weak and a large wavefunction component with the same ℓ-value is involved in the decay to the 2^+ state, a significant 2^+ branching ratio can be obtained. This is the case for the deformed emitter ^{131}Eu [7], illustrated in Figure 2, where a 2^+ branching ratio of 24(5)% was obtained.

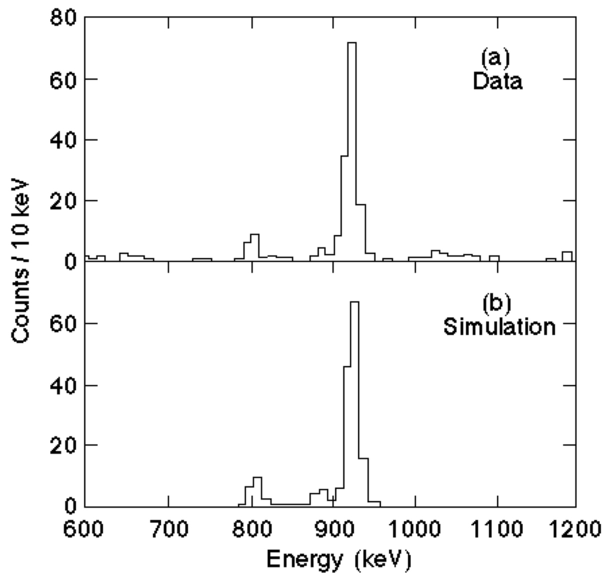

Figure 2: a) Observed energy spectrum of protons from the decay of ^{131}Eu. b) Simulated energy spectrum for protons from the decay of ^{131}Eu, assuming a branching ratio of 24(5)%.

Using the formalism described above, we have calculated the decay widths for the deformed proton emitters $^{131}\mathrm{Eu}(\frac{3}{2}^+)$, $^{141g}\mathrm{Ho}(\frac{7}{2}^-)$, and $^{141m}\mathrm{Ho}(\frac{1}{2}^+)$, as well as the branching ratios for the decays to the 2^+ states of the daughter nuclei ^{130}Sm and ^{140}Dy, respectively. The results are shown in Table 1. Good agreement with experiment is obtained. These calculations can be improved by the addition of pairing.

Table 1: Calculated and experimental proton decay widths (in units of 10^{-20} MeV). The 2^+ widths Γ_2 have been calculated with $E_{2^+} = 122$ keV for ^{130}Sm and $E_{2^+} = 202$ keV [8] for ^{140}Dy. S_{exp} is the experimental spectroscopic factor. Pairing is not included.

Nucleus	Γ_0	S_{exp}	Γ_2	$\frac{\Gamma_2}{(\Gamma_0+\Gamma_2)}$
^{131}Eu($3/2^+$)	2.88	-	0.929	0.244
Experiment [7]	1.71(24)	0.59	0.54(13)	0.24(5)
^{141g}Ho($7/2^-$)	15.0	-	0.10	0.007
Experiment [9]	10.9(10)	0.73	-	< 0.01
^{141m}Ho($1/2^+$)	24800	-	94.5	0.004
Experiment [10]	5700(2140)	0.23	-	-
Experiment [9]	7020(1080)	0.28	-	< 0.01

4 Near-Spherical Proton Emitters

The recent observation of fine structure in the proton decay of ^{145}Tm [11] suggests that the simple spherical approach does not provide a full description of the proton emission process for near-spherical nuclides. In this picture, the conservation of angular momentum only allows decay to the ground state, with no possibility of calculating decay to excited states of the daughter. In the case of ^{145}Tm, two proton groups were observed with the same half-life of 3.0(3) μs, one populating the ground state and the other populating the first 2^+ state of the daughter nucleus ^{144}Er at 0.33 MeV with a branching ratio of 9.6(15)% [11].

At first glance this suggests a deformed emitter, and we have used the formalism of Ref. [5] to calculate, in the adiabatic limit, the half-lives and 2^+ branching ratios for ^{145}Tm with ground-state spins $\frac{1}{2}^{\pm} \leq J \leq \frac{11}{2}^{\pm}$. Both prolate and oblate quadrupole deformations of $|\beta_2| = 0.18$ were used, as suggested by the excitation energy in ^{144}Er. Only the K=$\frac{5}{2}^-$ oblate deformed solution, with half-life of 2.3 μs and branching ratio of 21%, comes close to agreeing with the experimental half-life and branching ratio values of 3.0(3) μs and 9.6(15)% [11], with a spectroscopic factor of 0.74(9). However, this state is not expected to be at the Fermi level for $^{145}_{69}$Tm; Ferreira and Maglione [12] show that the oblate K=$5/2^-$ state is at the Fermi level for $^{151}_{71}$Lu. Although recent calculations [13-15] predict prolate rather than oblate deformation for the ground states of ^{145}Tm and its decay daughter ^{144}Er, our decay rate results are not compatible with the prolate possibility.

Besides the oblate-deformed J=$5/2^-$ solution, we now consider another alternative: ^{145}Tm and other near-spherical proton emitters may have a time-averaged spherical shape, but their wavefunctions contain other components due to particle-vibration coupling. A look at the low-lying energy levels of the even-even daughters of the odd-A proton emitters between $^{145}_{69}$Tm and $^{177}_{81}$Tl suggests that this assumption may be valid. In most cases the ratio of excitation energies $E_x(4^+)/E_x(2^+)$ lie in the range 2.0-2.4, characteristic of an anharmonic vibrator. This indicates that the interaction between the last proton and the core nucleus should include particle-vibration coupling to the first 2^+ state of the daughter nucleus.

The case of an odd-A J=$11/2^-$ proton emitter will serve as an example. In addition to the j=$11/2^- \otimes 0^+$ component, which can only decay to the daughter ground state, the parent wavefunction will contain 5 additional components arising from j=$7/2^- \otimes 2^+$ through j=$15/2^- \otimes 2^+$, all coupled to J=$11/2^-$. These latter components can only decay to the 2^+ state of the daughter, with the decay widths dependent on the amount of energy available, the size of these components, and the ℓ-values involved. In the next section we discuss the calculation of proton decay rates using the coupled-channels formalism with a quadrupole particle-vibration coupling. In Sec. 6 we present applications of the method to several proton emitters.

5 Coupled-Channels Approach with Particle-Vibration Coupling

In order to calculate the proton decay rate, we need to determine the wavefunction of a nucleus consisting of a single proton interacting with an even-even core. We consider only the 0^+ ground state and the first excited 2^+ state of the core, and, as in [5], search for narrow unbound resonances in this coupled system. Here, however, the average core shape is spherical, with vibrational coupling to the proton. As in [5], we expand the total wavefunction for a given total spin (I, M) of the system as

$$\Psi_{IM}(\mathbf{r}) = \sum_{\ell j \lambda} \frac{\phi^I_{\ell j \lambda}(r)}{r} |\ell(j\lambda)IM\rangle, \tag{4}$$

where

$$|\ell(j\lambda)IM\rangle = \sum_{m\mu} \langle jm\lambda\mu|IM\rangle |\lambda\mu\rangle |\ell jm\rangle \quad (5)$$

is the channel-spin wavefunction, obtained by coupling the single particle spin-angular wavefunctions $|\ell jm\rangle$ to the core wavefunction $|\lambda\mu\rangle$. When $\lambda = 0$, $\ell = \ell_o$ and $j = I$.

The single-particle potential for a proton interacting with a spherical nucleus includes the nuclear and spin-orbit interactions and the Coulomb potential,

$$V_{\rm sp}(r) = V_N(r) + V_{\ell s}(r)\ \ell \cdot \mathbf{s} + V_C(r). \quad (6)$$

Our parametrization is given in Appendix A of Ref. [5]. There we discussed the generalization to the case of an axially-symmetric deformed core nucleus. Here we consider a vibrational core nucleus and employ the total Hamiltonian

$$H = -\frac{\hbar^2}{2m_0}\Delta + V_{\rm sp} + H_{\rm vib} + \delta V_{\rm vib}, \quad (7)$$

where m_0 is the reduced mass and $H_{\rm vib}$ is the intrinsic vibrational Hamiltonian of the daughter nucleus, with eigenvalues E_0 for the 0^+ ground state and E_2 for the first excited 2^+ state. The last term is the coupling between the single-particle motion and the vibrational excitation. Further details may be found in ref. [16]; we quote only the results here.

6 Applications to Near-Spherical Proton Emitters

We have used the coupled-channels Green's function method [16] to calculate the proton decay rates of both odd-A and odd-odd spherical proton emitters whose even-even core nuclei display vibrational properties. Since we are dealing with nuclei having $68 < Z < 82$, the $1h_{11/2}$, $2d_{3/2}$, $3s_{1/2}$, and higher shell model orbitals are available for inclusion in the coupled equations. As described in ref. [16], the strength of the vibrational coupling term has been determined from experimental values for the excitation energy of the first 2^+ state in the core. We have not included calculations for odd-A ^{155,157}Ta, whose daughter nuclei ^{154,156}Hf do not appear to be vibrational. The single-particle potential parameters used are identical to those used to successfully describe the decay of the deformed proton emitters ^{131}Eu and ^{141}Ho [5]. This demonstrates for the first time that the same single-particle potential can be used to describe proton decay of both near-spherical and deformed nuclei.

As mentioned in Section 4, fine structure has been observed in the decay of ^{145}Tm, with a ground state half-life of 3.0(3) μs and a 2^+ branching ratio of 9.6(15)% [11]. Using particle-vibration coupling, the calculated half-life of 2.0 μs and 2^+ branching ratio of 9.9% are in remarkably good agreement with the experimental values, yielding a spectroscopic factor of 0.66(8). As expected, the decay width to the 2^+ state is mainly due to the ℓ=3 proton emission from the $7/2^- \otimes 2^+$ component of the wavefunction.

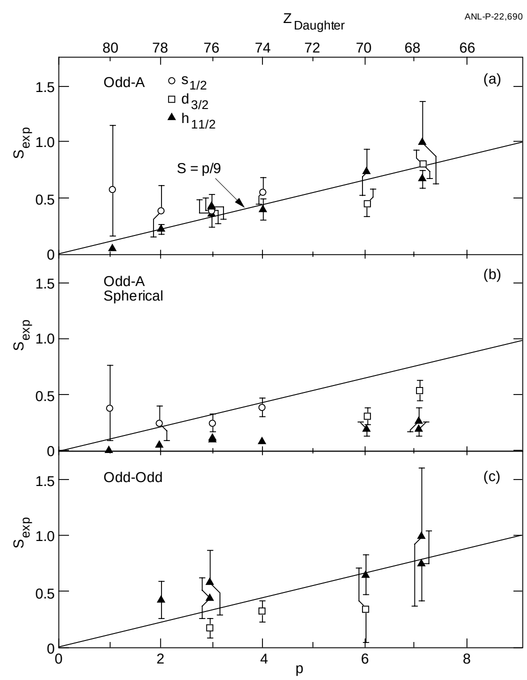

Figure 3: (a) Experimental spectroscopic factors S_{exp} calculated with particle-vibration coupling for the odd-A proton emitters, plotted as a function of p, the number of pairs of proton holes below Z=82 possessed by the daughter nucleus. The atomic number of the daughter nucleus Z_D is also shown, with $Z_D = 82 - 2p$. (b) Experimental spectroscopic factors for spherical odd-A proton emitters (particle-vibration coupling set to zero). (c) Same as (a) except for odd-odd proton emitters.

Fig. 3(a) shows the experimental spectroscopic factors S_{exp} for the odd-A proton emitters plotted as a function of p, the number of pairs of proton holes below Z=82 in the daughter nucleus. With the exception of ^{151}Lu^m and ^{177}Tl, they agree quite well with the low-seniority shell model calculation of spectroscopic factors described in Ref. [17], which is shown as the solid line in Fig. 3(a-c). For ^{177}Tl the $h_{11/2}$ and $s_{1/2}$ states are separated by 807 keV [18], and thus a spectroscopic factor calculated with the assumption of degenerate shell model orbitals will not be correct. The value of $S_{exp} = 0.46(12)$ for ^{151}Lu^m obtained here can be compared with the value of $0.34(^{+12}_{-8})$ obtained using a purely spherical approach [19]. In contrast to previous work [17, 20], the present value of S_{exp} for the other odd-A $d_{3/2}$ proton emitter ^{147}Tm^m agrees extremely well with the calculated value.

To demonstrate the importance of including particle-vibration coupling in the single particle potential, Fig. 3(b) shows the experimental spectroscopic factors calculated in a spherical picture for the same emitters as in Fig. 3(a). The agreement with the theoretical spectroscopic factors is now spoiled, clearly showing the important role played by particle-vibration coupling in the decay of near-spherical proton emitters.

For the odd-odd emitters, shown in Fig. 3(c), only the decay width to the daughter ground state is presented here. The unpaired neutron is considered to be a spectator in these calculations, and the assumed core nucleus is the even-even nucleus (Z-1,A-2). Again, we have not included calculations for ^{156}Ta, whose core nucleus ^{154}Hf does not appear to be vibrational.

7 Summary

Much progress has been made recently in calculating the decay rates for deformed and near-spherical proton emitters. The observation of fine structure in proton decay provides important clues on the wavefunction composition of proton emitters. Particle-vibration coupling is necessary to explain the observed decay rates for near-spherical emitters.

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