Advances in ANL Reactor Physics Methods Inspired by A. F. Henry

H. S. Khalil,[1)] T. A. Taiwo,[1)] W. S. Yang,[2)] P. J. Finck,[1)]

[1)] Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60439
[2)] Dept. of Nucl. Engineering, Chosun University, P. O. Box 275, Gwangju, 501-600, Korea

Reactor physics methods at ANL were traditionally developed primarily for fast-spectrum liquid metal reactor (LMR) design and analysis. The success of systematically derived nodal methods for light water reactor (LWR) analysis, to which A. F. Henry made essential contributions, motivated the development at ANL of analogous methods applicable to fast reactors and other systems. Henry's research and technical guidance[1-5] strongly influenced these methods. In this paper, we review the small part of Henry's legacy embodied in the ANL nodal methods for static, depletion, and transient reactor physics analyses.

In early 1980s, Lawrence developed the DIF3D nodal scheme[6] by extending the transverse integration nodal approach popularized by Henry[1] to hexagonal geometry. This scheme employs a nodal expansion method and the interface current formulation for solving the multigroup diffusion equations. Equations for the flux moments are obtained by applying the weighted residual method to the one-dimensional (1-D) flux equations resulting from the transverse integration procedure. The interface current equations are derived in a response matrix form using the source and higher-order leakage moments. This scheme is extremely effective for its primary application (fast reactors), its accuracy typically being similar to that of finite difference solutions employing 24 triangular mesh per a hexagon and a factor of ~5 finer axial mesh. The nodal scheme was implemented in the REBUS-3 fuel cycle analysis code[7] as one of the neutronics solution options and is routinely used for fast reactor design computations.

Motivated by the analysis needs resulting from the Integral Fast Reactor fuel cycle demonstration and other advanced LMR designs, a reconstruction method based largely on Henry's research[3] was developed for recovering LMR pin burnup characteristics from fuel cycle calculations performed in hexagonal-z geometry using the nodal diffusion option of the DIF3D/REBUS-3 code system.[8] Intranodal distributions of group fluxes, nuclide densities, power density, burnup, and fluence are efficiently computed using non-separable higher order polynomial shapes constrained to satisfy nodal information. Compared to fine-mesh reference solutions, the maximum errors in power and nuclide densities were less than 2% for driver assemblies and typically less than 5% for blanket assemblies. The calculated and measured values for local fuel pin burnups in IFR test assemblies in the Experimental Breeder Reactor II agreed within the combined experimental uncertainties. This reconstruction method is used extensively to characterize EBR-II spent fuel undergoing treatment at ANL-W.

To enable the nodal solution to match results of higher-accuracy models, a nodal equivalence scheme modeled after the scheme formulated by Smith and Henry,[2] was developed for hexagonal assemblies and implemented in the DIF3D nodal option.[9] For each hexagonal-z node and energy group, discontinuity factors are introduced for surface-averaged fluxes and partial currents and directional flux moments. These discontinuity factors are defined such that the reference nodal quantities weighted with discontinuity factors satisfy the nodal equations when the reference eigenvalue, node-averaged fluxes and net currents are conserved. It was verified that the use of reference discontinuity factors yields the reference solution. Systematic schemes for computing approximate discontinuity factors that reduce spatial truncation errors in large LMRs and reproduce transport effects in small LMRs were developed. These schemes were shown to significantly improve the accuracy of the nodal solutions for several three-dimensional benchmark problems.

Henry’s pioneering work in reactor kinetics strongly influenced the extension of the DIF3D nodal scheme to time-dependent problems.[10] The time-dependent nodal equations are solved with one of two major time discretization schemes: the theta method or the space-time factorization method. The theta method is a variable integration scheme that permits the resulting difference equations to range from fully explicit to fully implicit. The factorization method allows the use of the improved quasistatic, adiabatic, or conventional point kinetics option for treatment of the time dependence. In the improved quasistatic option, the approach of Henry and Kao[4] is adopted -- enabling the flux shape to be computed using the same algorithm employed for the fully implicit theta scheme. In all these factorization options, the flux amplitude is obtained from the solution of the point kinetics equations employing time-dependent kinetics parameters evaluated by the code. This method has been used to analyze a variety of thermal- and fast-spectrum systems.

In order to apply the nodal methods to perturbation theory calculations and kinetics calculations based on space-time factorization schemes, two techniques for solving the mathematical adjoint equations of the DIF3D nodal scheme were developed: the similarity transformation procedure and a direct solution scheme.[11] This work was motivated by previous work led by Henry at MIT.[5] The similarity transformation approach is based on a linear transformation of the physical adjoint flux moments. It produces the exact mathematical adjoint solution for the flat transverse leakage approximation and a good approximate solution for the quadratic leakage approximation. The direct solution scheme rigorously computes the mathematical adjoint solution for both flat and quadratic transverse leakage approximations. In this scheme, adjoint nodal equations are cast in a form very similar to that of the forward equations by employing a linear transformation of the adjoint partial currents. This enables the use of the forward solution algorithm with only minor modifications for solving the mathematical adjoint equations.

Motivated by the need to improve the accuracy of the transverse-integration nodal method in hexagonal-z geometry, particularly for thermal systems, the variational nodal transport code VARIANT,[12] that employs multidimensional expansions for the intra-nodal flux was developed. The variational nodal method is a hybrid finite element method that guarantees nodal balance and permits spatial refinement through the use of hierarchical complete polynomial trial functions. Angular variables are expanded with complete or simplified spherical harmonics up to order P_5 with full anisotropic scattering capability. A wide range of hexagonal-geometry benchmarks showed that the VARIANT diffusion option is consistently more accurate than the DIF3D nodal option or other diffusion methods previously applied to the same benchmarks. Mathematical adjoint solutions are obtained by employing an approach similar to the adjoint solution method for the DIF3D nodal option.[13] A transient analysis capability based on VARIANT was recently developed and shown to provide accurate results for dynamic problems with significant transport effects.[14]

In summary, A. F. Henry's research and mentorship have profoundly enhanced reactor analysis methods at ANL. In addressing remaining challenges, reactor analysts at ANL and elsewhere continue to benefit from the knowledge he imparted and from his systematic and creative approach to challenging problems.

References

1. G. Greenman, K. S. Smith, and A. F. Henry, "Recent Advances in an Analytic Nodal Method for Static and Transient Reactor Analysis," *Proc. Topl. Mtg. Computational Methods in Nuclear Engineering*, Williamsburg, Virginia, April 23-25, 1979, Vol. 1, p. 3-49 (1979).

2. K. S. Smith, A, F. Henry, and R. A. Lorentz, "The Determination of Homogenized Diffusion Theory Parameters for Coarse Mesh Nodal Analysis," *Proc. Conf. Advances in Reactor Physics and Shielding*, Sun Vally, Idaho, September 14-17, 1980, p. 294 (1980).

3. H. S. Khalil, P. J. Finck, and A. F. Henry, "Reconstruction of Fuel Pin Powers from Nodal Results," *Proc. Topl. Mtg. Advances in Reactor Computations*, Salt Lake City, Utah, April 28-30, 1983, Vol. I, p. 367 (1983).

4. P. Kao and A. F. Henry, "Supernodal Analysis of PWR Transients," *Proc. Conf. Advances in Reactor Physics and Shielding*, Santa Fe, New Mexico, April 9-13, 1989, Vol. 2, p. 63 (1989).

5. T. A. Taiwo and A. F. Henry, "Perturbation Theory Based on a Nodal Model," *Nucl. Sci. Eng.*, **92**, 34 (1986).

6. R. D. Lawrence, "The DIF3D Nodal Neutronics Option for Two- and Three-Dimensional Diffusion Theory Calculations in Hexagonal Geometry," ANL-83-1, Argonne National Laboratory (1983).

7. B. J. Toppel, "A User's Guide to the REBUS-3 Fuel Cycle Analysis Capability," ANL-83-2, Argonne National Laboratory (1983).

8. W. S. Yang, P. J. Finck, and H. Khalil, "Reconstruction of Pin Power and Burnup Characteristics from Nodal Calculations in Hexagonal Geometry," *Nucl. Sci. Eng.*, **111**, 21 (1992).

9. P. J. Finck and K. L. Derstine, "The Application of Nodal Equivalence Theory to Hexagonal Geometry Lattice," *Proc. Int. Topl. Mtg. Advances in Mathematics*, Computations, and Reactor Physics, Pittsburgh, Pennsylvania, April 28-May 2, 1991, Vol. 4, p. 16.14-1 (1991).

10. T. A. Taiwo and H. S. Khalil, "DIF3D-K: A Nodal Kinetics Code for Solving the Time-Dependent Diffusion Equation," *Proc. Int. Conf. Mathematics and Computations, Reactor Physics, and Environmental Analyses*, Portland, Oregon, April 30-May 4, 1995, Vol. 2, p. 1171 (1995).

11. W. S. Yang, T. A. Taiwo, and H. Khalil, "Solution of the Mathematical Adjoint Equations for An Interface Current Nodal Formulation," *Nucl. Sci. Eng.*, **116**, 42 (1994).

12. G. Palmiotti, E. E. Lewis, and C. B. Carrico, "VARIANT: Variational Anisotropic Nodal Transport for Multidimensional Cartesian and Hexagonal Geometry Calculation," ANL-95/40, Argonne National Laboratory (1995).

13. K. F. Laurin-Kovitz and E. E. Lewis, “Eigenvalue Perturbation Calculations for the Variational Nodal Method,” *Trans. Am. Nucl. Soc*., **70**, 200 (1992).

14. J. E. Cahalan, T. Ama, G. Palmiotti, T. A. Taiwo, and W. S. Yang, "Development of a Coupled Dynamics Code with Transport Theory Capability and Application to Accelerator-Driven Systems Transients," *Proc. of ANS International Topical Meeting on Advances in Reactor Physics and Mathematics and Computation into the Next Millennium, PHYSOR 2000*, Pittsburgh, Pennsylvania, U.S.A., May 2000.